BASIC 7

MATHEMATICS HANDOUT

FOR

1ST TERM 2022/2023 SESSION

COMPILED BY,

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SCHEME OF WORK.

WEEKS	CONTENTS
1	NUMBER SYSTEM: Numerals, Place value, Counting large numbers.
2	NUMBER SYSTEM: Fractions, ordering of fractions, equivalent fractions, conversion of fractions to Decimals and Percentages.
3	NUMBER SYSTEM: Factors and Multiples (HCF and LCM)
4	NUMBER SYSTEM: Estimation of Length, Mass, Volume and Capacity.
5	NUMBER SYSTEM: Approximation to Significant figures, decimal places, nearest placed values, Addition, Subtraction, Multiplication and Division of Approximated values.
6	NUMBER SYSTEM: Counting in Base Two and Denary Numbers, Conversion from one base to another.
7	NUMBER SYSTEM: Directed Numbers
8	NUMBER SYSTEM: Directed Numbers
9	BASIC ALGEBRA: USE OF SYMBOLS
10	BASIC ALGEBRA: USE OF SYMBOLS
11	Revision and Examination.

WEEK 1

TOPIC; NUMBER SYSTEM; Whole number

SPECIFIC OBJECTIVE; At the end of the class, the Students should be able to

i. Define and explain Numerals
ii. Translate the Roman numerals to ordinary Numbers and vice versa.
iii. Define and find the place value of Numbers.
iv. Use place value system to write out large numbers in words and vice versa.

NUMERALS

Numerals are symbols that represent numbers e.g Roman Numerals, Hindu Numerals, Arabic Numerals, Tally e.t.c.

Before we look at the Roman Numerals, we should first consider what Numbers are.

Numbers are mathematical objects used to count, measure or label.

Below are the types of Numbers;

1. Natural Numbers
2. Integers
3. Complex Numbers
4. Real Numbers
5. Rational and Irrational Numbers
6. Fractions

ASSIGNMENT

Write short note on all the types/ classification of numbers mentioned above.

<u>ROMAN NUMERALS</u>

Roman Numerals are the roman symbols used to represent numbers.

Numbers	**Roman Symbols**
1	I
2	Ii

3	Iii
4	Iv
5	V
9	Ix
10	X
20	Xx
40	xL
50	L
90	Xc
100	C
500	D
900	CM
1000	M
5000	$\bar{V}$
10000	$\bar{X}$

From the above table we have that converting from roman numerals to ordinary numbers, we use the following examples:

1. MCXII
2. CMLVII

<u>ANSWERS</u>

1. M =1000
   C = 100
   X = 10
   I = 1
   I = 1
   1112
2. Unlike the first example, in this case, we discover that a smaller number C is before a Bigger number M, we first subtract them i.e bigger – smaller
   1000 – 100 = 900 (M – C = CM)
   Having established that CM = 900, we proceed as in the case of number 1 example.
   CM = 900
   L = 50
   V = 5
   II = 2
   957

ASSIGNMENT

Translate the following Roman Numerals to Ordinary Numbers.

1. $\bar{X}MMDLVIII$
2. MMMMMDCC
3. CDXXXIV

PLACE VALUE SYSTEM

Placed value system is a system that helps to tell the position of a digit in a Number. Example, the place value of the numbers written boldly in the set of numbers below are;

1. 231**5**4 = **5 TENS (50)**
2. 2000**1** = **1 UNIT (1)**
3. **1**00000 = **1 HUNDREDS OF THOUSAND (100000)**
4. 200**3**456 = **3 THOUSAND (3000)**
5. 5.**2**3445 = **2 TENTH (2/10)**

ACTIVITY

What is the place value of the digits written in bold from the sets of numbers below?

1. 33**4**210000
2. 2000.00**5**23
3. **4**20000120
4. **6.**025350
5. **0.**0000**5**32

S/NO	PLACED VALUE	NUMERICAL ARRANGEMENTS
1.	Tenth	0.1
2.	Hundredth	0.01
3.	Thousandth	0.001
4.	Unit	1
5.	Tens	10
6.	Hundred	100
7.	Thousand	1 000
8.	Tens of Thousand	10 000
9.	Hundreds of Thousand	100 000
10.	Million	1 000 000

11.	Tens of Million	10 000 000
12.	Hundreds of Million	100 000 000
13.	Billion	1 000 000 000
14.	Tens of Billion	10 000 000 000
15.	Hundreds of Billion	100 000 000 000
16.	Trillion	1 000 000 000 000

COUNTING OF LARGE NUMBERS

In expressing large numbers in words, this can only be possible with the knowledge of Placed value system discussed earlier.

The following steps must be put into consideration.

1. Separate the numbers in group of threes (3) starting from the right to the left.
2. Count the numbers to indicate the place value of each digit in that number.

Consider the Example below.

1. 21309000
2. 3003003003003
3. 411050056

SOLUTION

1. First step: separating the numbers in group of threes from right to left we have

   21 309 000

   Second step: counting the numbers to indicate the place value of each digit
   2 = 2 Tens of Million = twenty million
   1 = 1 Million = One million
   3 = 3 Hundreds of Thousands = Three Hundred Thousands
   0 = 0 Tens of Thousands
   9 = 9 Thousand = Nine Thousand
   0 = 0 Hundred
   0 = 0 Tens
   0 = 0 Units

   Now, we can put them together saying:
   Twenty – One Million, Three Hundred and Nine Thousand.

ACTIVITY: Use the above approach and solve for the remaining two examples.

NOTE: Same approach in second step above is applied when expressing from words to ordinary numbers.

WEEK 2 COMMON FACTORS

Specific Objective: I should be able to:

a. Define and identify factors of numbers
b. Identify common factors and HCF of whole numbers
c. Define and identify multiples of numbers
d. Identify common multiples and LCM of numbers
e. Solve problems on HCF and LCM

A Common factor is a factor common to two or more numbers.For example, 5 is a factor of 10 and also a factor of 20 and 45. This means 5 is a common factor of 10, 20 and 45.

Note that there may be more than one common factor.

Example

Find the common factors of 20 and 24

Solution: First list out the factor of each number

20 = 1, 2, 4, 5, 10, 20

24 = 1, 2, 3, 4, 6, 8, 12, 24

The common factors are 1, 2, and 4

HIGHEST COMMON FACTOR (HCF)

The highest common factor of two or more numbers is the highest factor common to them.

From the example above, the common factors of 20 and 24 are 1, 2, and 4

Therefore, the HCF of 20 and 24 is 4 because it's the biggest factor.

ALTERNATIVE METHOD

2	20	**24**
2	10	**12**
	5	**6**

HCF = 2x2

=4

COMMON MULTIPLES

Multiples are numbers obtained when a whole number multiplies another whole number.
Multiples are simply numbers in the times table or beyond.
E.g Multiples of 4
4 = 4X1
8 = 4X2
12 = 4X3
16 = 4X4
20 = 4X5

The multiples of 4 include; 4,8,12,16,20….etc.

Common multiples are multiples common to two or more numbers.

Example:
The multiples of 3 are:
3,6,9,12,15,18,21,24,27,30,33,36…..etc.

The multiples of 4 are:
4,8,12,16,20,24,28,32,36,40,44…..etc.

The first two common multiples of 3 and 4 are 12 and 24

LOWEST COMMON MULTIPLES (LCM)

The lowest common multiple of two or more numbers is the lowest multiple they have in common.

For example, the common multiples of 3 and 4 as illustrated above are 12, 24, 36, etc. the smallest of these multiples is 12.Thus, 12 is the LCM.

Method 2

One can also find the LCM of a set of numbers by using the products of their prime numbers.

$3= 1x3 = 1^1 x3^1$

$4= 2x 2 =2^2$

Now select each prime factor with the highest power

The LCM = $3^1 \times 2^2$

$= 3 \times 4 = 12$

Find the LCM of 8,10 and 12

$8 = 2 \times 2 \times 2 = 2^3$

$10 = 2 \times 5 \quad = 2^1 \times 5^1$

$12 = 2 \times 3 \quad = 2^1 \times 3^1$

LCM = 2^3 X 5 X 3

LCM = 8 X 5 X 3

LCM = <u>120</u>

Activity

Find the LCM of the following

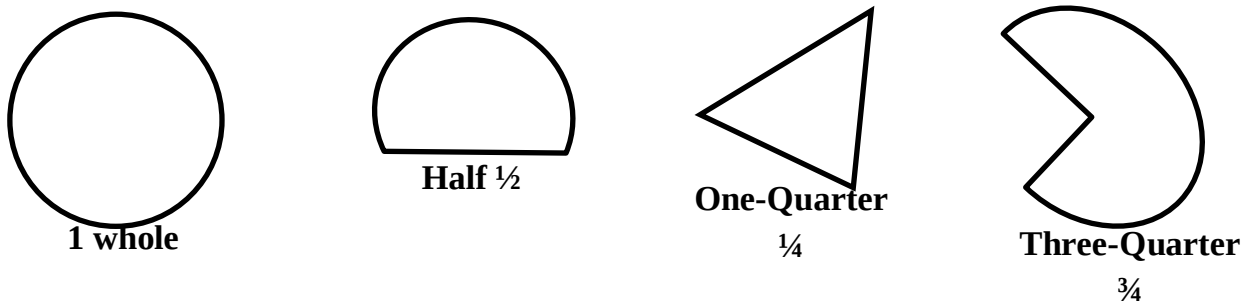
1) 4 and 6
2) 3, 6, 10 and 18
3) 18, 36 and 48

WEEK 3: Fractions

Specific Objective: I should be able to;

a. Identify equivalent fractions
b. Arrange fractions in order of size
c. Convert fractions to decimals and vice versa
d. Convert of fractions to percentages and vice versa
e. Add and subtract fractions

A Fraction is a portion or part of a whole for example



A fraction has two parts. The number on the top of the line is called the numerator. It tells how many equal parts of the whole or collection are taken the number below the line is called the denominator. It shows the total divisible number of equal parts which are there in a collection.

1 ⟵ Numerator
3 ⟵ Denominators

Fractions may be divided into common and decimal fractions

1. Common or vulgar fractions:-

Common fraction is always written as one number over another. E.g $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}$, etc.

Types of common fraction

1. **<u>Unit Fractions:</u>** Fractions with numerator 1 are called unit factors E.g.

$$^1/_2, ^1/_3, ^1/_4, ^1/_5 \text{ etc.}$$

2. **<u>Proper Fractions:</u>** Fractions in which the numerator is less than the denominator are called proper fractions E.g

$^1/_2, ^2/_3, ^4/_5, ^3/_7$ etc.

3. **<u>Improper Fractions:</u>** Fractions in which the numerator is more than or equal to the denominator are called improper fractions E.g $\frac{4}{4}$ $\frac{4}{3}$ $\frac{5}{4}$ etc.

4. **<u>Mixed Fractions:</u>** This consist of a whole number along with a proper fraction E.g $1\frac{1}{3}$, $3\frac{3}{4}$, $2\frac{2}{3}$, etc.

2. Decimal fractions:

Decimal fractions are simply called decimals e.g 0.82, 5.47, 7.02, 0.05, etc.

Equivalent fractions

Equivalent fractions are fractions that have the same value or fractions that are equal. E.g $\frac{1}{2}$ $=\frac{2}{4}$ $=\frac{3}{6}$ $=\frac{4}{8}$ are all equivalent fractions.

Finding equivalent fractions

To obtain an equivalent fraction, simply multiply or divide both the numerator and the denominator of the given fraction by the same number.

To obtain an equivalent fraction with larger numerators and denominators, simply multiply the numerator and denominator by the same number. For example:

To change 1/3 into 2/6 simply multiply the numerator and the denominator by 2. i.e 1/3 = 1/3 x 2/2 = 2/6.

To continue the sequence simply multiply by 3,4,5,6,7……. These numbers are called multipliers.

$\frac{1}{3}$ x $\frac{2}{2}$ = $\frac{2}{6}$ x $\frac{3}{3}$ = $\frac{6}{18}$ x $\frac{4}{4}$ = $\frac{24}{72}$

Hence: $\frac{1}{3}$ = $\frac{2}{6}$ = $\frac{6}{18}$ = $\frac{24}{72}$.

To obtain an equivalent fraction with smaller numerators and denominators. Simply divide both the numerator and denominator by the same number.

For example:

To change 6/18 into 2/6 simplify divide the numerator and the denominator by 3.

$\frac{6}{18} \div \frac{3}{3} = \frac{2}{6}$.

Example: Convert 2/9 into an equivalent fraction with the denominator 54

Solution:

2/9 = $\boxed{}$/54

Divide the second denominator by the first denominator to obtain the multiplier.

54÷9 = 6

$\frac{2}{9} \times \frac{6}{6} = \frac{12}{54}$

So $\boxed{}$=12

Ordering of fractions

Ordering of fractions is the arrangement of fractions in order of size.

Fractions with same denominators

When the denominators of fractions are equal (i.e the same), the one with the largest numerator is the largest fraction and the one with the smallest numerator is the least.

Example:

Arrange 4/5, 1/5, 6/5, 3/5 in ascending order.

Note: Ascending order means arrangement from the smallest to the biggest.

1/5, 3/5, 4/5, 6/5 (in ascending order).

Fractions with different denominators

In arranging fractions with different denominators, first find the LCM of all the denominators.

Example:

Which is greater: 2/3 or 4/5?

Solution

Look for the LCM of the denominators i.e the LCM of 3 and 5 is 15

Now write each fraction with the same denominator

(i.e 15) $\frac{15 \div 3 \times 2,\ 15 \div 5 \times 4}{15}$

$\frac{5 \times 2,\ 3 \times 4}{15}$

10/15, 12/15

This means 2/3 = 10/15 and 4/5 = 12/15.

Comparing the size of the numerators of

10/15 and 12/15 then:

4/5 is greater than 2/3.

Converting Fractions to decimals
In converting fractions to decimals and vice versa, one can make use of long division method.

Example:
Convert 3/4 to decimal

Solution:
3/4 = 0.75
4 | 3.00

Converting Decimals to Fraction :
Examples:
Convert 0.4 to fraction in its lowest term.

Solution:
1 digit after the decimal point (i.e I.D.P means divide by 10).
0.4 = 4/10 = 2/5

Converting Fraction to percentages:
A change of fraction or decimal to a percentage, multiply the fraction or the decimal by 100.
But when converting a percentage to a decimal or fraction, divide by 100.

Examples:
Express 14 as percentage

Solution
14 = 14 X 100% = 25%

Converting Percentages to Fractions

To convert a percentage to a percentage to a fraction, divide by 100.

Example:
Change 5% to fraction in its simplest form

Solution:

$5\% = \frac{5}{100} = \frac{1}{20}$

Addition and subtraction of fractions

Same denominators

To add or subtract fractions with the same denominators simply add or subtract the numerators and then put the result over the denominator.

For example:

Find the value of:

a. 5/8 – 3/8
b. 1 + 5/6

Solution

a. 5/8 – 3/8 = 5-3÷8 = 2/8 = ¼
b. 1+5/6 = 6/6 + 5/6 = 6+5/6 = 11/6 - $1\frac{5}{6}$

(Note: 6/6 =1)

Notice that 1+5/6 is simply $1\frac{5}{6}$

Different denominators

To add or subtract fractions with different denominators, first find the LCM of the denominators.

Example:

Simplify the following fractions:

a. 3/5 + ½
b. 2/5 + ½ +1/4

a. The denominators are 5 and 2

The LCM of 5 and 2 is 10

$\frac{(10 \div 5 \times 3) + (10 \div 2 \times 1)}{10}$

$$\frac{(2x3) + (5x1)}{19}$$

$$\frac{(2x3) + (5x1)}{10}$$

6+5 = 11/10 = 1/10

b. The denominators are 5, 2 and 4

The LCM of 5, 2 and 4 is 20

$$\frac{(20 \div 5x2) + (20 \div 2x1) - (20 \div 4x1)}{20}$$

$$\frac{(4x2) + (10x1) - (5x1)}{20}$$

$$\frac{8+10-5}{20}$$

$$\frac{18-5}{20} = \frac{13}{20}$$

Activity

Find the value of the following

a. $4\frac{5}{9} + 1\frac{1}{9}$
b. 1/5 + ½
c. ¼ + ½ - 3/8

WEEK 4 TOPIC; NUMBER SYSTEM

SUB TOPIC; ESTIMATION OF LENGTH, MASS, VOLUME AND CAPACITY.

SPECIFIC OBJECTIVE; At the end of the Class, the Students should be able to;

- **i.** Define and estimate the values of Length.
- **ii.** Define and estimate the values of Mass.
- **iii.** Define and estimate the values of Volume and Capacity.

ESTIMATION OF LENGTH

Length is a measure of the distance between two points. The Standard units of measurement of Length (SI UNIT) is Metres (m).

The unit of lengths from the smallest to greatest are;

10 Millimetres (mm) = 1 Centimetre (cm)

10 Centimetres (cm) = 1 Decimetre (Dm)

10 Decimetres (Dm) = 1 Metre (m)

1000 Metres (m) = 1 Kilometre (Km)

EXAMPLES

1. How many 20cm is found in 2m?
2. If Adam travelled 12km to School, by how many 120m has he travelled?

SOLUTION

1. Recall,

   100cm = 1m

   Therefore,

   200cm = 2m

Then,

The number of 20 cm found in 200cm = $^{200}/_{20}$

= 10.

2. Recall,

   1000m = 1km

Therefore,

12000m = 12km

Then,

Adam travelled 120m of 12000m = $^{12000}/_{120}$

= 100 times

ESTIMATION OF MASS

Mass is the measure of the amount of matter present in the body. The standard unit of measurement (SI Unit) is Kilogram (kg).

The Unit of measurement for Mass are,

10 Milligram (mg) = 1 Centigram (cg)

10 Centigrams (cg) = 1 Decigram (Dg)

10 Decigrams (Dg) = 1 Gramme (g)

1000 Grammes (g) = 1 Kilogram (Kg)

1000 Kilograms (Kg) = 1 Tonne (Ton)

Examples

Convert the following masses to grams

a. 2.1kg
b. 750mg

Solution

a. 1000g = 1kg

:. 2.1kg = 2.1 x 1000 = 21000g

Alternative method:

1000g = 1kg

☐ = 2.1kg

We don't know the value of 2.1kg in g that is why an empty box is used to represent the unknown. You cross multiply so that:

☐X 1 = 1000 x 2.1

☐ = 21000g

(Anything multiplied by 1 is same as that thing)

:. 2.1kg = 2100g

b. 100mg – 1g

:. 750mg = 750/1000 = 3/4 – 0.75g

Alternatively:

1000mg = 1g

750mg = $\square$

When you cross multiply

1000 x $\square$ = 750 x 1

1000 $\square$ = 750

Divide both sides by 1000 in order to find the value of the box.

$$\frac{1000}{1000}\ \square = \frac{750}{1000}$$

$\square$ = 750/1000

$\square$ = 3/4 , :. $\square$ 0.75g

:. 750mg = 0.75g

ESTIMATION OF CAPACITY

The capacity of a container is the amount of fluid (liquid) it can hold. The basic unit of capacity in metric system is the litre (L). The common units are litres, millitres (ML), and centiliters (CL).

CAPACITY

10 millilitires (ml)	= 1 centilitre (cl)
1000 centilitres	= 1 litre (l)
1000 militires	= 1 litre
1000 litres	= 1 kilolitres (kl)

1000 celitres = 1kilioters

1000 000 milllitres = 1 kilolitres

Examples

Convert 5cl to

(a) Ml (b) Kl

a. 10ml = 1cl

:. 5cl = 10 x 5 = 50 ml

Alternative method

10ml = 1cl

$\boxed{}$ = 5cl

Cross multiply so that

$\boxed{}$ x 1 = 10 x 5

$\boxed{}$ = 50ml.

:. 5cl = 50ml

b. 100 000cl = 1kl

:. 5cl = $\frac{5}{100\,000}$ = $\frac{1}{20\,000}$ = 0.00005kl.

<u>Activity</u>

1. Convert 5km to (a) m (b) mm
2. Convert these masses to milligrams (a) 0.08g (b) 0.0004kg
3. Convert the following capacities to litres (a) 721ml (b) 850cl (c) 1250ml

WEEK 5 **APPROXIMATION**

<u>Specific objective:</u> I should be able to:

a. Define approximation
b. Round off numbers
c. Approximate values of addition, subtraction, multiplication and division

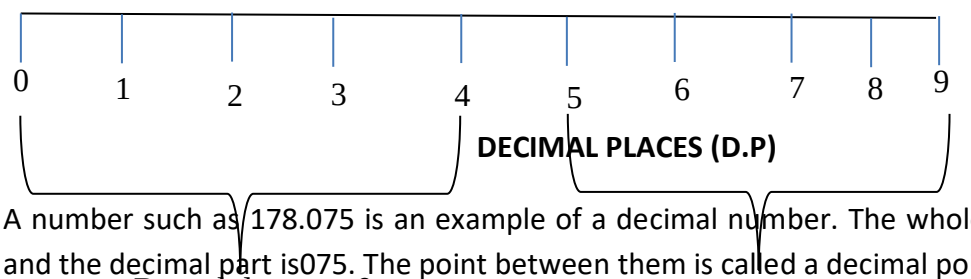
Approximation is a way of using rounded numbers to estimate the outcome of calculations. Approximation can help us decide whether an answer to a calculation is of right size (i.e. Magnitude) or not.

When approximating, one need to round each number usually to 1.S.f or the nearest whole number. Then use the rounded number for calculation.

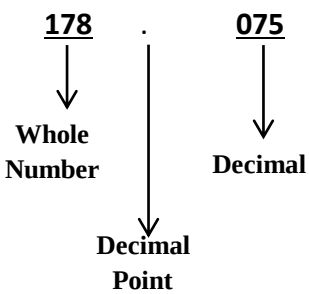
Note that approximation is not used to find an accurate answer to a calculation but to work out a rough answer that is close to the accurate answer.

The symbol ≈ means is approximately equal to.

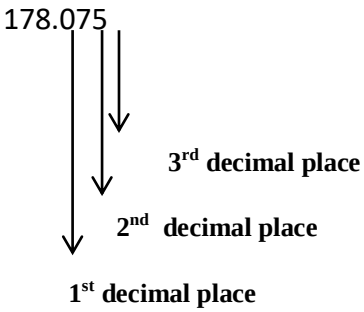
Rounding of Numbers



A number such as 178.075 is an example of a decimal number. The whole number part is 178 and the decimal part is075. The point between them is called a decimal point.



To find the decimal place (D.P) simply count number of figures after the deimal point. Thus, this number has 3d.p



Example

To Round a decimal number to a given number of decimal places

1. Look for the last digit (i.e. the required decimal place you are rounding to)
2. Then look at the next digit to the right, i.e., the decider.
3. If the decider is 5 or above, round up (i.e. add 1 to the last digit) if it is less than, add nothing

Example

Give the number 35.4782 Correct to:

1. 1 d.p 2. 3d.p

<u>Solution</u>

a) $35.\underline{\mathbf{4}}\overset{+1}{7}82$ = 35.5(1 d.p)

b) $35.47\underline{\mathbf{8}}2$ = 35.478(3 d.p)

<u>Rounding Decimals To The Nearest Tenth, Hundredth And Thousandth</u>

To round a decimal to the nearest tenth is same as rounding to 1 d.p likewise, hundredth and thousandth is same as rounding a number to 2d.p and 3d.p.

Example

Give 507.6757 correct to the nearest

a) Tenth
b) Hundredth
c) Thousandth

Solution

a) $507.\overset{\text{T}}{\underline{\mathbf{6}}}\overset{\text{H}}{7}\overset{\text{TH}}{5}\ 7$ = 507.7

b) $507.6\overset{\text{T}}{\underline{\mathbf{7}}}\overset{\text{H}}{5}\overset{\text{TH}}{7}$ = 507.68

c) $507.67\overset{\text{T}}{\underline{\mathbf{5}}}\overset{\text{H}}{7}\overset{\text{TH}}{}$ = 507.676

Rounding decimal to the nearest whole numbers

To round a decimal number to the nearest whole number look at the figure in the 1std.p, if it is 5 or more round the whole number up, if it is less than 5 do not change the whole number.

Example

Round off 42.4739 to the nearest whole number

<u>42</u>.4739 = 42.0000 =42

SIGNIFICANT FIGURE (S.F)

The word significant mean 'Important' and it is another way approximating numbers. A figure's position in a number shows what the figure is worth

For example, the number 40726 has 5s.f

40726

5thS.f

4thS.f

3rdS.f

2ndS.f

1stS.f

Also the number 40007000 has 4s.f

400700

4thS.f

3rdS.f

2ndS.f

1stS.f

Note that zero in between whole numbers are significant while zeros at the end are not significant.

Number less than zero

The number 0.0006430 has 3s.f

0.006430

3rdS.f

2ndS.f

1stS.f

The zero before the decimal number means that there are no units and the three zeros after the decimal point means that there are no tenths, no hundredth and no thousandths so the four zeros are not significant figures. Therefore the most significant figure is 6, then 4 and lastly 3.

Note that the first significant figure is always the first non-zero figure as you read a number from the left.

Example

Give 45775 correct to

(a) 1s.f (b) 3s.f

a. $\overset{+1}{\underline{\mathbf{4}}}5775$ =50,000

b. $45\overset{+1}{\underline{\mathbf{7}}}75$ = 45,800

Approximation Involving Addition and Subtraction

Addition and Subtraction

Example

Round off each number to 1s.f, then approximate the answers

a. $\underline{\mathbf{3}}07 - 116 \approx 300 - 100 = 200$

b. $\underline{\mathbf{6}}7.5 + \underline{\mathbf{9}}3.6 \approx 70 + 90 = 160$

Multiplication and Division

Example

Find the approximate answer to (i) 24.17 ÷ 2.65 (ii) 43 x 95

i. 24.17 nearest whole number is 24

2.65 nearest whole number is 3

24 ÷ 3 = 8

ii. 43 x 95

43 = 40(1s.f)

95 =100(1s.f)

40 x 100=4,000

Week 6: Counting in base 2

Specific objective:

a. Add and subtract two or three 3 digit binary numbers
b. Convert base ten numbers to binary numbers and other bases
c. Convert base two numbers and other bases to base ten number

Number Bases

The usual system of counting today is called the decimal or denary system. The denary or decimal system is also called base ten. This system enables us to be able to write small or large numbers using the combination of the digits i.e 0,1,2,3,4,5,6,7,8,9. For example: 6967, 678.56, 0.000942.

Apart from base ten, we sometimes count in other bases such as base two, base five, base eight, etc.

Binary system

In binary system, the greatest digit used is 1, so the two digits available in binary system are 0 and 1.

Binary system is also called base two

Arithmetic operations

Addition and subtraction of binary numbers

Addition

To add in base two note that:

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 10

Remember that in base 10, 1+1 =2 but 2 in base 2 is 10. So in binary, 1+1 = 10.

Example

Add 111_{two} to 11_{two}

Solution

$$\begin{array}{ccc} 1 & 1 & \\ 1 & 1 & 1 \\ & 1 & 1 \\ \hline 10 & 1 & 0_{two} \end{array}$$

$\longrightarrow$ numbers carried forward.

Explanation:

List column: 1 + 1 = 10 write down 0 and carry 1 forward

2nd column: 1 (carried) + 1+1 = 11. Write down 1 and carry 1 forward.

3rd column: 1 (carried) + 1 = 10

Subtraction:

To subtract in base two note that:

0 – 0 = 0

10 – 0 = 1

1 – 0 = 1

11 – 1 = 10

Example:

Subtract 1011_{two} from 10101_{two}

Solution

$$\begin{array}{c} 1\,0\,1\,0\,1 \\ \underline{1\,0\,1\,1} \\ 1\;\;0\;\;1\;0_{two} \end{array}$$

Explanation:

1st column: 1 – 1 = 0

2nd column: 0 – 1 is not possible, so borrow 1 from the next column to give 10.

So, 10 – 1 = 1

3rd column: 0 – 0 = 0

4th column: 0 – 1 is not possible, so borrow 1 from the next column to give 10. So, 10 – 1 = 1.

i.e $10101_{two} - 1011_{two} = 1010_{two}$

Activity

Calculate the following binary numbers

a. 10 + 11
b. 1101 + 1101
c. 1111 – 110
d. 1011 + 10 = 111

Puzzle

11 + 11, 111 + 111, etc. patterns.

11 + 11	= 110
111 + 111	= 1110
1111 + 1111	= 1110
1111 + 1111	= 11110

Using the above pattern, complete the following

11111 + 11111	=	
111111 + 111111	=	
1111111 + 1111111	=	
111111111 + 11111111	=	

<u>Converting numbers in base ten to base two</u>

Using the method of repeated division.

To convert from base ten to base two.

a. Divide the given base ten repeatedly by the two number
b. Continue dividing until zero is obtained, writing down the remainder at every stage
c. The answer is the remainder read upwards (i.e. from bottom to top).

Example:

Convert 184_{ten}, to a base two number

2	184
2	92 R 0
2	46 R 0
2	23 R 0
2	11 R 1
2	5 R 1
2	2 R 1
2	1 R 0
	0 R 1

$\therefore 184_{ten} = 10111000_{two}$

Note: You can extend the above ideas to convert numbers in base ten to other number bases.

For example:

Convert 405_{ten} to base 8

Solution

8	405	
8	50	R 5
8	6	R 2
	0	R 6

$\therefore 405_{ten} = 625_{eight}$

Activity

Convert the following base 10 numbers to the base stated

a. 30 to base 4
b. 207 to base 5
c. 140 to base 2

Converting numbers in other bases to base ten

To convert any number base to base ten, simply express it as the power of the given base.

Examples:

1. Express 10111_{two} in base ten

Solution

Using the expanding method.

$\overset{4\,3\;\;2\;1\;0}{10111}_{two}$ $(1x2^4) + (0x2^3) + (1x2^2) + (1x2^1) + (1x2^0)$

$(1x16) + (0x8) + (1x4) + (1x2) + (1x1)$

$16+0+4+2+1$

$= 23$

$\therefore 10111_{two} = 23_{two}$

2. Convert 534_{six} to base ten

Solution

$5^2\ 3^1\ 4^0{}_{six}$ $(5x6^2) + (3x6^1) + (4x6^0)$

$(5x36) + (3x6) + (4x1)$

$180 + 18 + 4 = 202_{ten}$

$\therefore 534_{six} = 202_{ten}$

Note that any letter or number with a power zero is equal to 1.

Activity

Convert the following to base ten

a. 45_{eight}
b. 3032_{four}
c. 20620_{eight}

WEEK 7 AND 8: DIRECTED NUMBERS (ADDITION AND SUBTRACTION)

Specific objectives: I should be able to:

a. Identify integers and directed numbers
b. Use number line in addition and subtraction of integers
c. Add and subtract integers without the use of number line

<u>Integers</u>

Integers are whole numbers greater than zero or less than zero and including zero itself.

Positive numbers are numbers greater than 0 and negative numbers are numbers less than 0.

Examples of integers: 4, 2, 0, 1, 5, etc.

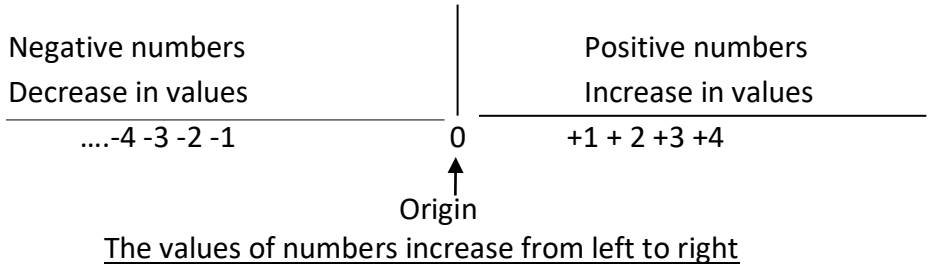
Positive and negative numbers are called directed numbers.

A negative number is always written with the ‘ – ‘ place in front of it. E.g -7, -10, -6, etc.

A positive number is usually written with a ‘+’ sign or no sign placed in front of it. Eg. +4 means 4 and +10 means 10.

<u>The Number line</u>

One can represent directed numbers on a number line as shown below:-



<u>The values of numbers increase from left to right</u>

Looking at the number line, you will observe that the numbers increase in values as you move from left to the right of the scale.

For example:

-3 is on the right of -4, so -3 is greater than -4. This means -4 is less than -3.

Note: > means is greater than e.g -3 >4

< means is less than e.g. -4 < -3

Similarly on the number line:

-1 is on the right of -6, so -1 > -4 or -4 < -1

4 is on the right of -12, so 4 > -12 or -12<4.

On a number line, a number to the right is always greater than a number to the left.

<u>Addition and subtraction of directed numbers using number line</u>

In a horizontal number line:

a. When adding, start from the first number, count right along the number line
b. When subtracting, start from the first number, count left along the number line

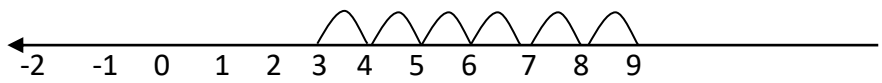
Example:

Use number line to find the value of:

a. 3+6 (b) -5-4

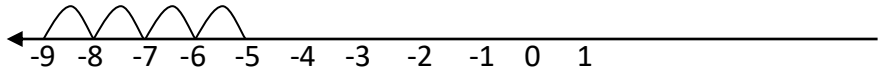
<u>Solution</u>

a. Start at 3, count 6 places to the right to end up at 9



So 3+6 = 9

b. Start at -5, count 4 places to the left to end up at -9



So -5 -4 = 9

<u>Addition and subtraction of directed numbers without the use of number line</u>

One can carry out addition and subtraction of directed numbers without drawing the number line.

<u>Rules:</u>

1. When you have two same signs, you add the numbers and then attach the common sign to the result.
   For example: +5 + 5 = 10
   -5 – 7 = - 12

   <u>Explanation</u>

a. The two fives are both positive values, so when they are summed up, a positive result is gotten. 5+5 = +10

b. Add the number together: 5+7 = 12. Now place a – sign in front of the answer to obtain -12 because both numbers have a negative sign in common.

-5 -7 – 12.

2. When you have two numbers with different signs subtract the smaller number from the larger number and then place the sign before the larger number in front of your answer.

For example: +3 – 9

<u>Solution</u>

The smaller number is 3, its sign is + and the larger number is 9, its sign is -

Subtract 3 from 9

i.e 9-3 = 6

Then place minus sign (i.e the sign before the larger number) in your answer to obtain -6.

:. + 3 – 9 = -6.

Also note that in addition and subtraction of directed numbers:

a. Replace the same signs that appear together by a positive sign.

   + + = +

   - - = -

   i. e.g +7 + (+8) = 7+8=15

      Same signs gives +

   ii. – 8 – (-6) = -8 +6 = -2

      Same signs gives +

b. Replace two different signs that appear together by a negative sign.

   + - = -

   - + = -

e.g (i) +6 + (-4) = 6 - 4 = 2

Different signs gives -

ii. 2 - + 6 = 2 – 6 - - 4

Different signs give –

Activity

1. Solve the following using number line
    a. 9 – 6
    b. – 5 – 6
    c. – 4 – 5 +12
2. Solve the following without the use of number line
    a. -12 -6
    b. 15 – 56
    c. -5 + (-7)
    d. – 7 – (6) -4

WEEK 9 AND 10: BASIC ALGEBRA: USE OF SYMBOLS

<u>Specific objective:</u>

I should be able to

a. Identify open sentences
b. Solve open sentences with two (+, -) arithmetic operations
c. Use letters to represent symbols or shapes in open sentences

Algebra is the branch of mathematics in which letters of the alphabets and symbols are used in place of numbers.

<u>Open sentence</u>

This is a statement that contains at least one blank or unknown and that becomes true or false when the blank is filled. E.g $\square$+3 = 8

Look carefully at the following mathematical statements:

a. 5 + 3 = 8 true
b. 6 x 3 = 12 false

We say (a) is true because 5 + 3 = 8. On the other hand we say (b) is not true because 6 x 3 – 18.

For example:

1. What number, when added to 4 gives 9?

   To answer this question, we represent the unknown number by any shape such as $\square$. Then this question maybe written in an open sentence like this.

   $\square$ + 4 = 9

   To make the open sentence true 5 must replace the box. i.e 5 + 4 = i9, :. $\square$ = 5

2. Find the missing numbers that make the following open sentences true:
   a. $\square$+ 7 = 14
   b. 30 + $\square$ - 9

<u>Solution</u>

$\square + 7 = 14$ Check:

$\square = 14 - 7$ $7 + 7 = 14$

$\square = 7$

Note that when a positive number i.e 7 crosses over an equal sign, the positive sign changes to a negative sign. i.e -7.

$30 = \square - 9$

$30 + 9 = \square$

$39 = \square$

$\therefore \square = 39$

Check $30 = 39-9$

Activity

Find the missing numbers that make the following open sentences true:

$12 = 17 - \square$

$27 - 20 = \square$

$\square - 9 = 20$

<u>Using letters to represent numbers</u>

For example: instead of writing

$\square + 2 = 12$, we could write $x + 2 = 12$.

This means $\square$ is replace by x.

The statement $x + 2 = 12$ is called an algebraic sentence or equation. It means x plus two equals twelve. Thus for the statement to be true the value of x must be 10.

The use of letters to represent numbers was developed by an Arab Mathematician Muhammad Ibn Musa al-Khwarizimi. The word “Algebra” comes from the title of his book. “Al-Kitab Al-Jabr wa’l muqabala which was written in AD 847.

Algebra is very useful to express mathematical relationships in form of formulae.

Examples:

Find the value of each letter.

a. $x = 3 + 6$
b. $12 + e = 18$

Solution

a. $x = 3+6$
$x = 9$

b. $12 + e = 18$
$e = 18 – 12$
$e = 6.$ $\therefore 12 + 6 = 18.$

Activity

1. Find the value of each letter
   a. $x – 14 = 2$
   b. $g – 7 = 10$
   c. $27 = x + x + x$

WORD PROBLEM

A man earns N50,000 in a month and his wife earns NZ. If the couple earn 95,000 altogether in a month, what is the value of Z?

Solution

The man earns = N50,000

The wife earns = NZ

Sum of their earning – N95,000

The value of Z will be;

N50,000 + NZ = N95,000

NZ = N95,000 – N50,000

NZ = N45,000

Check: N50,000 + N45,000 = N95,000

:. Z = N45,000

Activity:

1. A student goes shopping with NP. He spent N5,000. If he now has N400, what is the value of P?

N.B; REVISION QUESTIONS ARE FROM THE BECE PAST QUESTIONS ON ALL TOPICS TAUGHT THROUGH THE TERM.