BASIC 9

MATHEMATICS HANDOUT

FOR

1ST TERM 2022/2023 SESSION

COMPILED BY...

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SCHEME OF WORK.

WEEKS	CONTENTS
1	**LOGIC; Logical reasoning (Simple and Compound Statements)**
2	**EUCLIDEAN GEOMETRY; Construction of Triangles with given sides, bisection of angles 30^0, 45^0, 60^0, 90^0.**
3	**EUCLIDEAN GEOMETRY; Locus of moving points equidistance from two points, two lines, and constant distance from a point.**
4	**LINEAR ALGEBRA; Matrices (Multiplication of Matrices, Rotation of Unit Square, Reflection, Displacement).**
5	**LINEAR ALGEBRA; Matrices (Shearing, Magnification and multiple mapping).**
6	**EUCLIDEAN GEOMETRY; Proof of some basic theorems (sum of angles in a triangle, The exterior angle of a Triangle equals sum of two interior opposite angles, Angles of Parallel Lines, Angles in a Polygon, Congruent Triangles, Properties of a Parallelogram, Intercept theorem).**
7	**EUCLIDEAN GEOMETRY; Mensuration (Length of an Arc. Perimeter of a Sector and Segment of a Circle, Area of Sector and Segment of a Circle, Relationship between the Sector of a Circle and the surface area of a Cone).**
8	**EUCLIDEAN GEOMETRY; Mensuration (Surface area and volume of Solid Shapes : Cubes, Cuboids, Cylinder, Cone, Prism, pyramids, Sphere, Hemisphere, Frustum, Compound Shapes.).**
9	**TRIGONOMETRY; Trigonometric Ratio of special angles**
10	**TRIGONOMETRY; Trigonometric ratios related to Unit Circles, Graphs of Sine and Cosine of Angles.**
11	**REVISION**

WEEK 1

TOPIC; LOGIC; LOGICAL REASONING

SPECIFIC OBJECTIVE; At the end of the class, the Students should be able to

  i.   **Define and explain the concept of logic.**
  ii.   **Define and Differentiate a Simple Statement from a Compound statement.**
  iii.   **Identify the Logical connectives for a Compound Statement.**
  iv.   **Translate a logical statement to logical symbols and vice versa.**
  v.   **Construct the Truth table for Logical statements**

LOGIC.

Logic is the branch of philosophy that is mainly concerned with the evaluation of arguments based on fundamental laws of scientific reasoning and thinking.

The word Logic is derived from the Greek word "LOGOS" which means "Thought, Reason or Law". It deals with the ability to argue and convince.

STATEMENT

There are basically two types of statements which are;

  i.   **Simple Statements;** this is a sentence that contains an idea and can be either True or False. Some examples of Simple statements are written below;
  a) Ada is a Boy. (this is certainly False because, the name Ada is feminine)
  b) Abuja is the Capital of Nigeria. (This is True because, it is)
  c) Nigeria is in Europe. (This is False be cause, Nigeria is an African country)

N. B; Not every Sentence is a Statement but, every Statement is a Sentence.

Example of such Sentences that are not Statements Are Interrogative sentences, Exclamation, Optative Sentences and Imperative Sentences e. t. c

The examples of Simple statements above are also called "**Logical Simple Statements".**

CLASS EVALUATION

1. Write out 10 examples of Logical Simple Statements.
2. Write 5 examples each of Imperative, interrogative, Optative and exclamatory Sentences.

TRUTH VALUE OF A STATEMENT

The Truth Value of a statement is the Truth or Falsity of that statement. This can be easily determined when there is a pre – knowledge and or definition of the concept behind the statement.

CLASS EVALUATION/ EXAMPLE

State the Truth value of the following statement below

1. $2 + 5 > 9$
2. $34 < 5 \times 4$
3. A Trapezium is a Rectangle.
4. The Diagonals of a Square bisect each other at 90^0.
5. A Rhomboid has no lines of Symmetry.
6. There are 14 vertices in a hexagonal Prism.
7. The area of a Triangle is πr^2.
8. A Circle has just one Diameter.
9. An equilateral triangle has no equal sides.
10. 33 is a prime number.

NEGATION OF LOGICAL STATEMENTS

If a statement is denoted by "P", then the negation is "Not P'. it is represented as ($\sim P$ or P').

This can be shown using a truth table. **A Truth table** is a table that shows the Truth or falsity of a logical statement.

P	$\sim P$
T	F
F	T

ii.   **Compound Statements;** These statements are formed when two or more Simple Logical statements with more than one idea are connected together by a certain group of words called Logical connectives to form a Sentence. Examples of such statements are;

a) Mark is fat and joy is slim.
b) She is not intelligent or does not study.
c) Abuja is the capital of Nigeria and a city in Africa.

LOGICAL CONNECTIVES

These are words used to connect two or more Simple statements to form Compound statements.

S/NO	Logical Connectives	Word	Notation
1.	Conjunction	And	∧
2.	Disjunction	Or	V
3.	Negation	Not	~
4.	Implication	Implies	→
5.	Bi - implication	If and only if	↔
7.	Equivalence	Is	≡

N. B; for every compound statement, there is always an ANTECEDENT and the CONSEQUENT part. Where the ANTECEDENT is the first statement while the CONSEQUENT is the last statement.

TRANSLATING LOGICAL STATEMENTS TO LOGICAL SYMBOLS

Consider the examples below,

1. John is Tall and handsome.

SOLUTION

Let, John is Tall be **P.**

John is Handsome be **Q.**

The Logical connective there is **AND** which is noted as ∧

Therefore, the statement can be interpreted as **P ∧ Q.**

Now, do the following.

2. If Pastors talk preach then, they talk a lot.
3. John can speak English fluently if and only if He was born and raised in England.
4. Either Ada or John came in late last night.
5. Lawyers are liars then Ali is a liar.

LOGICAL CONNECTIVES AND THEIR TRUTH TABLES

1. **CONJUNCTION;** in the case of conjunction of two or more simple statements joined together with the "AND" word having the symbol indicated in the last table above, "**The statement is True when all simple statements are true otherwise, its False.**"
Considering the truth table below for Conjunction.

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

2. **DISJUNCTION;** in this case, two or more simple statements joined together with the word "OR" having the symbol as stated in the table for logical connectives above, **"The statement is True if one or more of the simple statements are true otherwise, its False."**
Considering the truth table below;

P	Q	P v Q
T	T	T
T	F	T
F	T	T
F	F	F

3. **IMPLICATION (CONDITIONAL STATEMENT);** in a conditional statement, "**It is completely False when the Antecedent (First statement) is True and the Consequent (Last statement) is False otherwise, it is True.**"

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

4. **BI – IMPLICATION (BI – CONDITIONAL);** in this case, **"The compound statement is True when both simple statements are either True or False otherwise, it is False."**

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

N. B; The formulae to indicate the number of rows a compound statement should have is denoted as 2^n

Where n = The number of simple statements in the compound statement.

WEEK; 2 AND 3
TOPIC; EUCLIDEAN GEOMETRY (CONSTRUCTION) PRACTICAL CLASS
SPECIFIC OBJECTIVE; At the end of the class, the Student should be able to:
i.   **Bisect angles 30^0, 45^0, 60^0, 90^0.**
ii.   **Construct Triangles with given sides.**
iii.   **Define and explain Locus.**
iv.   **Construct Locus of moving points equidistance from two points, two lines, and constant distance from a point.**

<u>**BISECTION OF SPECIAL ANGLES**</u>

In construction, from the previous classes, students have been taught how to construct special Angles such as **60^0, 90^0.**

We shall be looking at bisecting these angles.

1. Bisecting angle **90^0** gives 45^0 and 135^0.
2. Bisecting 60^0 gives 120^0 and 30^0.

The practical session will be done in class by the Teacher.

N. B; EVERYTHING ABOUT CONSTRUCTION WILL BE DONE IN CLASS AS A PRACTICAL SESSION.

WEEK; 4 AND 5

TOPIC; LINEAR ALGEBRA (MATRICES)

SPECIFIC OBJECTIVE; At the end of the class, the Student should be able to;

i.   **Solve Multiplication of Matrices.**
ii.   **Solve Rotation of Unit Square, Reflection, Displacement.**
iii.   **Solve Shearing, Magnification and multiple mapping of matrices.**

<u>MATRIX</u>

This is a rectangular array of real numbers arranged in rows and columns. There are m – rows and n – columns which together forms an (m x n) Matrix order.

$$\begin{bmatrix} a11 & \cdots & a1n \\ \vdots & \ddots & \vdots \\ am1 & \cdots & amn \end{bmatrix}$$

<u>TYPES OF MATRIX</u>

1. Singular Matrix
2. Rectangular Matrix
3. Square Matrix
4. Unit Matrix
5. Diagonal Matrix
6. Null Matrix

Assignment; Write short notes on the above mentioned types of Matrices.

<u>MULTIPLICATION OF MATRICES</u>

Given the example for a 2 x 2 Matrix below,

A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and B = $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

A x B = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ x $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

$$= \begin{pmatrix} (a \: x \: e) + (c \: x \: f) & (b \: x \: e) + (d \: x \: f) \\ (a \: x \: g) + (c \: x \: h) & (b \: x \: g) + (d \: x \: h) \end{pmatrix}$$

LET'S LOOK AT THE EXAMPLES BELOW FOR A X B

1.   A = $\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & -3 \\ 4 & 3 \end{pmatrix}$

2. $A = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$ $B = \begin{pmatrix} -4 & -1 \\ 1 & 3 \end{pmatrix}$

SOLUTION.

1. $A \times B = \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & -3 \\ 4 & 3 \end{pmatrix}$

$$= \begin{pmatrix} (-2 \ x \ 2) + (0 \ x -3) & (1 \ x \ 2) + (3 \ x -3) \\ (-2 \ x \ 4) + (0 \ x \ 3) & (1 \ x \ 4) + (3 \ x \ 3) \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -7 \\ -8 & 13 \end{pmatrix}$$

NOW SOLVE THE OTHER EXAMPLE YOURSELF.

MATRICES AS LINEAR TRANSFORMATION

Transformation is a mapping between two plane or Solid figures.

Considering the point $P_1 = \begin{pmatrix} x1 \\ y1 \end{pmatrix}$ is to be a linear mapping of another point $P = \begin{pmatrix} x \\ y \end{pmatrix}$ if a set of linear algebraic equations as shown below

x1 = ax + by   …………………………………………….i

y1 = cx + dy   …………………………………………….ii

the above equations can be written as;

$$\begin{pmatrix} x1 \\ y1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

The above is called Transformation matrix.

MAPPING; this is a relationship between a set X and another set Y in which each element of X has one element of Y.

ROTATION; given the centre of rotation C, the line joining any point P to C can be rotated through any fixed angle. Angles of rotation are measured positive in the anti – clockwise direction.

A rotation through 360^0 about the Origin brings a point P back to P. a rotation through 180^0 about the Origin is called a half turn.

Considering $P\begin{pmatrix} x \\ y \end{pmatrix}$, it will rotate and produce an image through 180^0 as $\begin{pmatrix} -x \\ -y \end{pmatrix}$.

SHEARING; this involves the addition of a multiple of one row or column to another. Such a matrix is derived by taking the identity matrix and replacing one of the zero elements with a non – zero value

For example, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Above is an identity matrix, replacing one of the zeros with a non – zero we have;

$\begin{pmatrix} 1 & ʌ \\ 0 & 1 \end{pmatrix}$. This is a typical Shear Matrix, and it shears parallel to the x – axis.

Where ʌ = A non – zero value.

SOME WORK EXAMPLES.

Given A = $\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}$ such that A mapped to (1, 1)

Solution

For A to be mapped to (1,1) implies that,

$$\begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} (1\,x - 2) + (1\,x\,1) \\ (1\,x\,0) + (1\,x\,3) \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

WEEK; 6

TOPIC; EUCLIDEAN GEOMETRY (PLANE SHAPES)

SPECIFIC OBJECTIVE; At the end of the class, the Student should be able to;

i.   **Prove some basic theorems in Triangles.**
ii.   **Solve Angles of Parallel Lines.**
iii.   **Solve Angles in a Polygon.**
iv.   **Solve Congruent Triangles.**
v.   **List the properties of a Parallelogram.**
vi.   **Prove the Intercept theorem.**

TRIANGLES

A triangle is the smallest polygon which has three sides and three interior angles. In this article, we are going to discuss the angle sum property and the exterior angle theorem of a triangle with its statement and proof in detail.

Angle Sum Property of a Triangle Theorem

In the given triangle, ΔABC, AB, BC, and CA represent three sides. A, B and C are the three vertices and ∠ABC, ∠BCA and ∠CAB are three interior angles of ΔABC.

Angle Sum Property of a triangle

Theorem 1: Angle sum property of triangle states that the sum of interior angles of a triangle is 180°.

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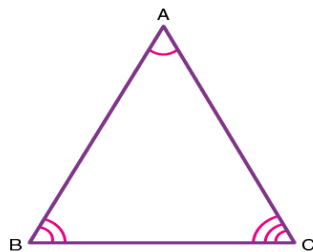
Proof:

Consider a ΔABC, as shown in the figure above. To prove the above property of triangles, draw a line parallel to the side BC of the given triangle.

Angle sum property of a triangle theorem 1
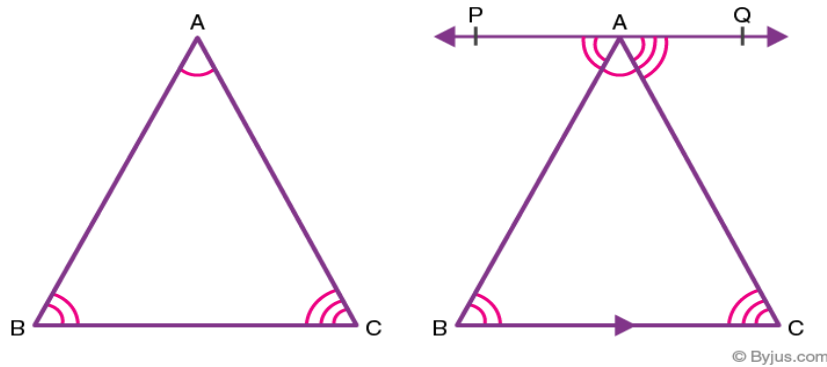
Since PQ is a straight line, it can be concluded that:

∠PAB + ∠BAC + ∠QAC = 180° ……(1)

Since PQ||BC and AB, AC are transversals,

Therefore, ∠QAC = ∠ACB (a pair of alternate angle)

Also, ∠PAB = ∠CBA (a pair of alternate angle)

Substituting the value of ∠QAC and ∠PAB in equation (1),

∠ACB + ∠BAC + ∠CBA= 180°

Thus, the sum of the interior angles of a triangle is 180°.

Exterior Angle Property of a Triangle Theorem

Theorem 2: If any side of a triangle is extended, then the exterior angle so formed is the sum of the two opposite interior angles of the triangle.
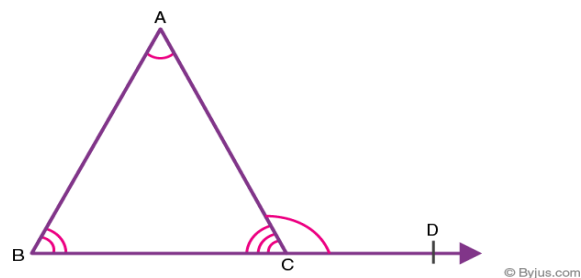
Angle sum property of a triangle theorem 2

In the given figure, the side BC of ΔABC is extended. The exterior angle ∠ACD so formed is the sum of measures of ∠ABC and ∠CAB.
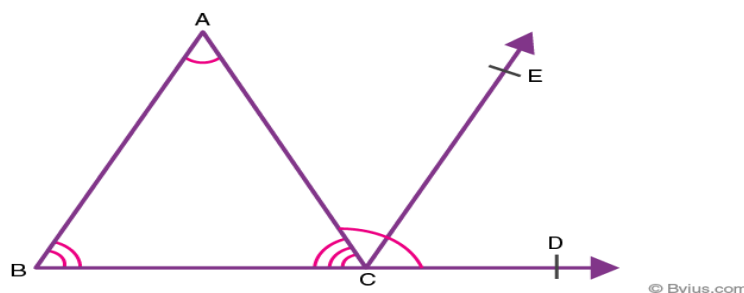
Proof:

From figure 3, ∠ACB and ∠ACD form a linear pair since they represent the adjacent angles on a straight line.

Thus, ∠ACB + ∠ACD = 180° ……….(2)

Also, from the angle sum property, it follows that:

∠ACB + ∠BAC + ∠CBA = 180° ……….(3)

From equation (2) and (3) it follows that:

∠ACD = ∠BAC + ∠CBA

This property can also be proved using the concept of parallel lines as follows:

Angle sum property of a triangle theorem 2 proof In the given figure, side BC of ΔABC is extended. A line parallel to the side AB is drawn, then: Since and is the transversal,

∠CAB = ∠ACE   ………(4) (Pair of alternate angles)

Also,

And is the transversal

Therefore, ∠ABC = ∠ECD……….(5) (Corresponding angles)

We have, ∠ACB + ∠BAC + ∠CBA = 180° ………(6)

Since the sum of angles on a straight line is 180°

Therefore, ∠ACB + ∠ACE + ∠ECD = 180° ………(7)

Since, ∠ACE + ∠ECD = ∠ACD(From figure 4)

Substituting this value in equation (7);

∠ACB + ∠ACD = 180° ………(8)

From the equations (6) and (8) it follows that,

∠ACD = ∠BAC + ∠CBA

Hence, it can be seen that the exterior angle of a triangle equals the sum of its opposite interior angles.

POLYGONS

Polygons are 2 – Dimensional shapes with at least 3 sides.

S/No	NAME OF POLYGON	NUMBER OF SIDES	TOTAL INTERIOR ANGLES $= (n - 2) \times 180$
1.	Trigon (Triangles)	3	180
2.	Quadrilateral	4	360
3.	Pentagon	5	540
4.	Hexagon	6	720
5.	Heptagon	7	900
6.	Octagon	8	1080
7.	Nonagon	9	1260
8.	Decagon	10	1440
9.	Dodecagon	12	1800

CONGRUENT TRIANGLES

Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.

We can tell whether two triangles are congruent without testing all the sides and all the angles of the two triangles. In this lesson, we will consider the four rules to prove triangle congruence. They are called the SSS rule, SAS rule, ASA rule and AAS rule. In another lesson, we will consider a proof used for right triangles called the Hypotenuse Leg rule. As long as one of the rules is true, it is sufficient to prove that the two triangles are congruent.

The following diagrams show the Rules for Triangle Congruency: SSS, SAS, ASA, AAS and RHS. Take note that SSA is not sufficient for Triangle Congruency. Scroll down the page for more examples, solutions and proofs.

Rules for Triangle Congruency

SSS

SAS

ASA

AAS

RHS

SSA is not sufficient for congruency. It may make two different triangles.
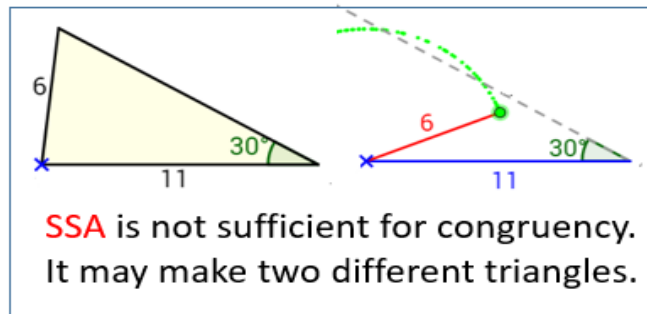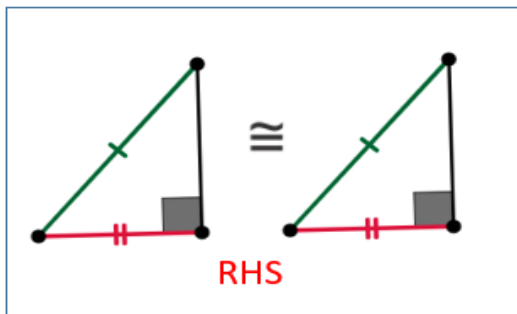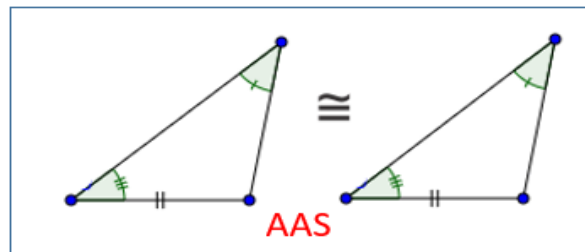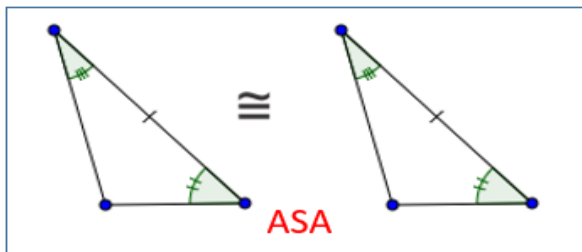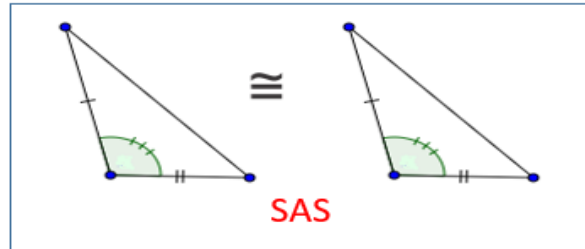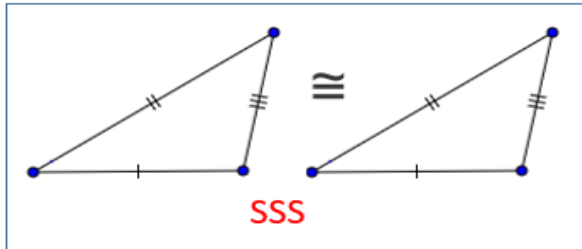
WEEK; 7 AND 8.

TOPIC; EUCLIDEAN GEOMETRY (MENSURATION)

SPECIFIC OBJECTIVE; At the end of the class, the Student should be able to;

i.   **Solve for the Length of an Arc.**
ii.   **Solve the Perimeter of a Sector and Segment of a Circle, Area of Sector and Segment of a Circle.**
iii.   **Deduce the relationship between the Sector of a Circle and the surface area of a Cone.**
iv.   **Solve the surface area and volume of Solid Shapes: Cubes, Cuboids, Cylinder, Cone, Prism, pyramids, Sphere, Hemisphere, Frustum, Compound Shapes.**

ARCS, SECTORS AND SEGMENTS OF A CIRCLE

A circle has always been an important shape among all geometrical figures. There are various concepts and formulas related to a circle. The sectors and segments are perhaps the most useful of them. In this article, we shall focus on the concept of a sector of a circle along with area and perimeter of a sector.

What is a Circle?

A circle is a locus of points equidistant from a given point located at the centre of the circle. The common distance from the centre of the circle to its point is called the radius. Thus, the circle is defined by its centre (o) and radius (r). A circle is also defined by two of its properties, such as area and perimeter. The formulas for both the measures of the circle are given by;

Area of a circle = πr^2

The perimeter of a circle = $2\pi r$

What is Sector of a circle?

The sector is basically a portion of a circle which could be defined based on these three points mentioned below:
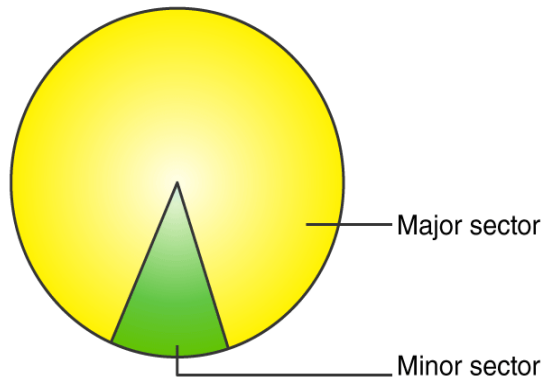
A circular sector is the portion of a disk enclosed by two radii and an arc.

A sector divides the circle into two regions, namely Major and Minor Sector.

The smaller area is known as the Minor Sector, whereas the region having a greater area is known as Major Sector.
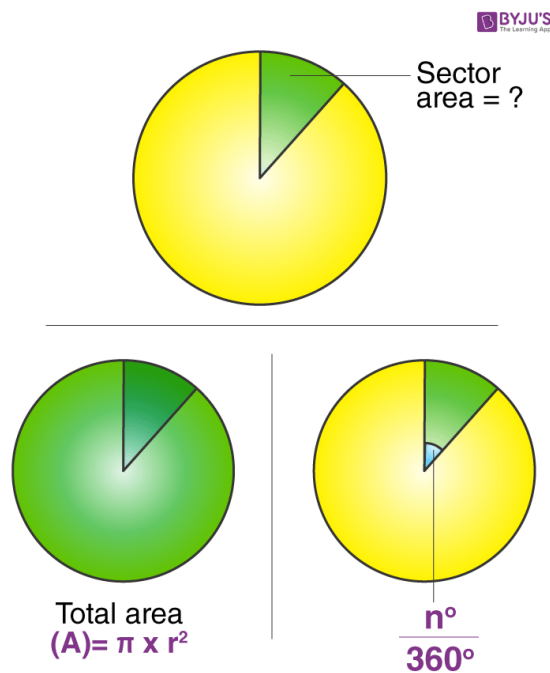
Sector: Major and Minor Sector

SECTOR OF A CIRCLE

Major sector

Minor sector

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Area of a sector

In a circle with radius r and centre at O, let ∠POQ = θ (in degrees) be the angle of the sector. Then, the area of a sector of circle formula is calculated using the unitary method.

Sector area = ?

Total area
(A)= π x r²

$\dfrac{n°}{360°}$

For the given angle the area of a sector is represented by:

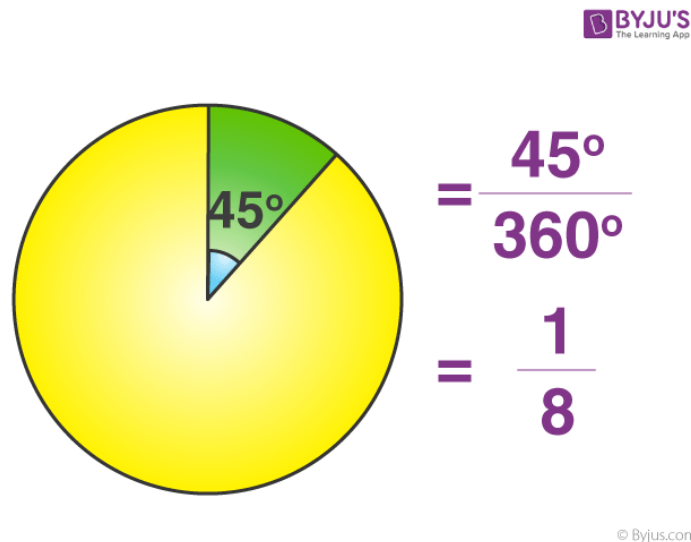The angle of the sector is 360°, area of the sector, i.e. the Whole circle = πr2

When the Angle is 1°, area of sector = $\pi r^2/360°$

So, when the angle is θ, area of sector, OPAQ, is defined as;

A = (θ/360°) × πr²

Total area of sector

Let the angle be 45° therefore, the circle will be divided into 8 parts, as per the given in the below figure;

$$= \frac{45°}{360°}$$

$$= \frac{1}{8}$$

Parts of Sector of a Circle

Now the area of the sector for the above figure can be calculated as (1/8) (3.14×r×r).

Thus the Area of a sector is calculated as:

A = (θ/360) × 22/7 × r^2

Length of the Arc of Sector Formula

Similarly, the length of the arc (PQ) of the sector with angle θ, is given by;

l = (θ/360) × 2πr

 (or)

l = (θπr) /180

Area of Sector with respect to Length of the Arc

If the length of the arc of the sector is given instead of the angle of the sector, there is a different way to calculate the area of the sector. Let the length of the arc be l. For the radius of a circle equal to r units, an arc of length r units will subtend 1 radian at the centre. Hence, it can be concluded that an arc of

length l will subtend l/r, the angle at the centre. So, if l is the length of the arc, r is the radius of the circle and θ is the angle subtended at the centre, then;

θ = l/r, where θ is in radians.

When the angle of the sector is 2π, then the area of the sector (whole sector) is πr2

When the angle is 1, the area of the sector = πr2/2π = r2/2

So, when the angle is θ, area of the sector = θ × r2/2

A = (l/r) × (r2/2)

A = (lr)/2

Examples

Example 1: If the angle of the sector with radius 4 units is 45°, then find the length of the sector.

Solution:

 Area = (θ/360°) × πr2

= (45°/360°) × (22/7) × 4 × 4

= 44/7 square units

The length of the same sector = (θ/360°) × 2πr

l = (45°/360°) × 2 × (22/7) × 4

l = 22/7

Example 2: Find the area of the sector when the radius of the circle is 16 units, and the length of the arc is 5 units.

Solution:

If the length of the arc of a circle with radius 16 units is 5 units, the area of the sector corresponding to that arc is;

A = (lr)/2 = (5 × 16)/2 = 40 square units.

Area of Sector of a Circle

Area of a Sector of Circle Formula

Area of a Segment of a Circle Formula

Segment and Areas of Segment of a Circle

Parts of a Circle

Perimeter of a Sector

The perimeter of the sector of a circle is the length of two radii along with the arc that makes the sector. In the following diagram, a sector is shown in yellow colour.

Perimeter of a sector

The perimeter should be calculated by doubling the radius and then adding it to the length of the arc.

Perimeter of a Sector Formula

The formula for the perimeter of the sector of a circle is given below:

Perimeter of sector = radius + radius + arc length

Perimeter of sector = 2 radius + arc length

Arc length is calculated using the relation:

Arc length = l = (θ/360) × 2πr

Therefore,

Perimeter of a Sector = 2 Radius + ((θ/360) × 2πr)

Example

A circular arc whose radius is 12 cm, makes an angle of 30° at the centre. Find the perimeter of the sector formed. Use π = 3.14.

Solution:

Given that r = 12 cm,

θ = 30°

= 30° × (π/180°) = π/6

Perimeter of sector is given by the formula;

P = 2 r + r θ

P = 2 (12) + 12 (π/6)

P = 24 + 2 π

P = 24 + 6.28

   = 30.28

Hence, Perimeter of sector is 30.28 cm

Practice Questions

A sector is cut from a circle of radius 21 cm. The angle of the sector is 150o. Find the length of the arc, perimeter and area of the sector.

A pizza with 21 cm radius is divided into 6 equal slices (slices are in the shape of a sector). Find the area of each slice.

The minute hand of a clock is 7 cm long. Find the area swept by the minute hand in 35 minutes.

Surface Area of Cone

The surface area of a cone is the amount of area occupied by the surface of a cone. A cone is a 3-D shape that has a circular base. This means the base is made up of a radius or diameter. The distance between the center of the base and the topmost part of the cone (of course, in the case of ice cream, this portion is at the bottom) is the height of the cone. We can find the surface area of the cone in two ways - total surface area and curved surface area of cone.

In this article, we will learn how to calculate the surface area of a cone in this article. The total surface area includes both curved and flat circular area whereas the curved surface area includes the area of only the curved surface. We will go through the formula and solved a few examples for a better understanding of the concept.

What is the Surface Area of Cone?

The area occupied by the surface/boundary of a cone is known as the surface area of a cone. It is always measured in square units. Stacking many triangles and rotating them around an axis gives the shape of a cone. As it has a flat base, thus it has a total surface area as well as a curved surface area. We can classify a cone as a right circular cone or an oblique cone. The vertex in the right circular cone is usually vertically above the center of the base whereas the vertex of the cone in an oblique cone is not vertically above the center of the base.

Surface Area of Cone Formula

As a cone has a curved surface, thus we can express its curved surface area as well as total surface area. A cone has two kinds of surface area:

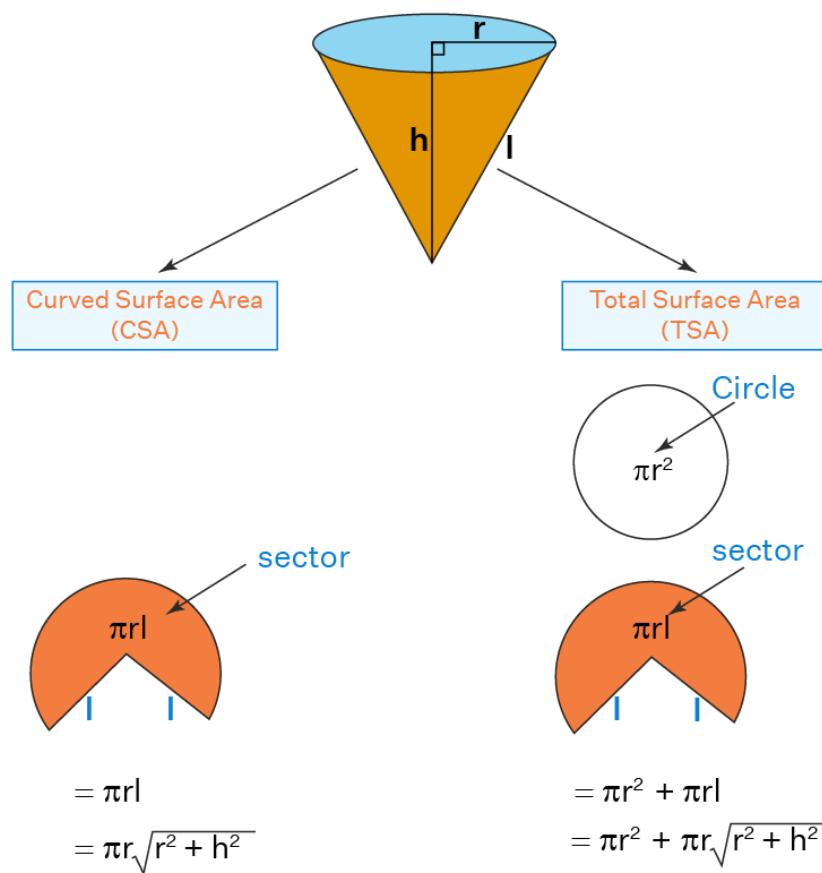Total Surface Area

Curved Surface Area

If the radius of the base of the cone is "r" and the slant height of the cone is "l", the surface area of a cone is given as:

Total Surface Area, T = πr(r + l) square units

Curved Surface Area, S = πrl square units

surface area of cone formula

Surface Area of Cone Formulas

By applying Pythagoras theorem on the cone, we can find the relation between the surface area of the cone and its height. We know, h2 + r2 = l2 where h is the height of the cone, r is the radius of the base, and l is the slant height of the cone.

⇒

l = √ (h2 + r2)

Thus,

The total surface area in terms of height can be given as, T = πr (r + l)

T = πr (r + √ (h² + r²)).

The curved surface area of the cone in terms of height can be given as S = πrl = πr(√(h2 + r2)).

Curved Surface Area of Cone

We discussed the formula to find the curved surface area of cone, let us now understand its meaning. Like other three-dimensional shapes, the cone also has both flat and curved surfaces. The curved surface area of cone refers only to the curved part of the cone which is other than the circular flat base. To find the curved surface area of cone, we multiply the radius and slant height of the cone by pi(π). Let us derive the formula for the curved surface area of cone below.

Curved Surface Area of Cone Formula

The curved surface area of the cone can be given by finding the area of the sector by using the formula,

Area of the sector (in terms of length of arc) = (arc length × radius)/ 2 = ((2πr) × l)/2 = πrl.

∴ The curved surface area of a cone, S = πrl units2.

Derivation of Surface Area of Cone

Let us take a cone of height "h", base radius "r", and slant height "l". In order to determine the surface area of cone derivation, we cut the cone open from the center which looks like a sector of a circle (a plane shape).

surface area of cone derivation

The total surface area of cone = area of the base of cone + curved surface area of a cone

Total surface area of cone = πr2 + πrl

= πr (r + l).

∴ The total surface area of cone, T = πr (r + l) units²

Finding Surface Area of Cone

Let us solve an example below to understand the application of the total surface area of cone and the curved surface area of cone.

Example:

 Find the total surface area and curved surface area of the cone whose radius is 7 inches and slant height is 3 inches. (Use π = 22/7).

We know, the total surface area of the cone is πr (r + l), and the lateral surface area of a cone is πrl. Given that: r = 7 inches

l = 3 inches

and π = 22/7.

Thus, total surface area of cone,

T = πr (r + l)

= (22/7) × 7 × (7 + 3)

= (22/7) × 7 × 10

= 22 × 10

= 220 in^2.

∴ The total surface area of the cone is 220 in^2.

The curved surface area of the cone,

S = πrl

= (22/7) × 7 × 3

= 66 in^2.

∴ The curved surface area of the cone is 66 in^2.

WEEK; 9 AND 10.

TOPIC; TRIGONOMETRY.

SPECIFIC OBJECTIVE; At the end of the class, the Student should be able to;

i.   **Solve trigonometric Ratio of special angles.**
ii.   **Solve trigonometric ratios related to Unit Circles.**
iii.   **Plot Graphs of Sine and Cosine of Angles.**

Trigonometric Ratios of Some Specific Angles

Definition

Trigonometric ratios of some specific angle are defined as the ratio of the sides of a right-angle triangle with respect to any of its acute angles. Trigonometric ratios of some specific angles include 0°, 30°, 45°, 60° and 90°. Now, let us learn how to find the trigonometric ratios of these angles in detail.

Trigonometric Ratios of 45°

Consider a right-angle triangle ABC as shown in the figure.

Right Triangle ABC
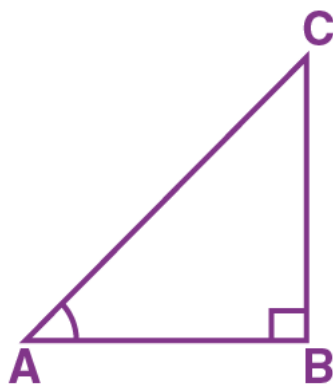
Here, a triangle ABC is right-angled at B. (i.e) ∠B = 90°.

If one of the angles is 45°, then the remaining angle should be 45°, as the sum of the interior angles of a triangle is 180°.

Therefore, ∠A = ∠C = 45°.

Also, AB = BC.

Now, assume that AB = BC = a

By using Pythagoras theorem, we can say that $AC^2 = AB^2 + BC^2$

$AC^2 = a^2 + a^2$

$AC^2 = 2a^2$

Therefore, AC = a√2.

Therefore,

sin 45° = BC/AC = a/ a√2 = 1/√2.

cos 45° = AB/AC = a/ a√2 = 1/√2.

tan 45° = BC/AB = a/a = 1

As, cosecant, secant and cotangent are the reciprocal of sine, cosine and tangent, respectively, then we can write:

cosec 45° = 1/ sin 45° = √2

sec 45° = 1/ cos 45° = √2

Cot 45° = 1/ tan 45° = 1.

Trigonometric Ratios of 30° and 60°

Consider an equilateral triangle ABC. We know that in an equilateral triangle all the angles should be equal to 60°. Therefore,

∠A = ∠B = ∠C = 60°.

W   :ular line
Al
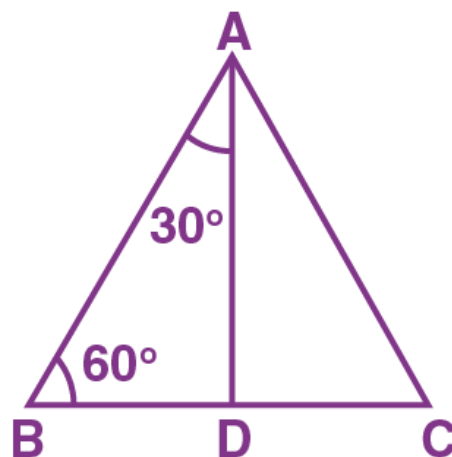
Equilateral triangle ABC

So, we can observe that, Δ ABD = Δ ACD

Also, BD = DC.

Hence, by using the CPCT (Corresponding Parts of Congruent Triangles) rule, we can write ∠ BAD = ∠ CAD.

So, the triangle ADB is a right triangle, which is right-angled at D and with ∠ BAD = 30° and ∠ABD = 60°.

To find the trigonometric ratio, we need to know the measurements of the side length of a triangle.

Assume that, AB = 2a.

Also, BD = ½ BC = ½ (2a) = a (Since, all the sides are equal in equilateral triangle)

Now, by using Pythagoras theorem, $AB^2 = BD^2 + AD^2$

Therefore, $AD^2 = AB^2 - BD^2$

$AD^2 = (2a)^2 - a^2$

$AD^2 = 4a^2 - a^2$

$AD^2 = 3a^2$

Hence, AD = a√3.

So, the trigonometric ratios of 30° are:

sin 30° = BD/AB = a/2a = ½

cos 30° = AD/AB = a√3 / 2a = √3/2

tan 30° = BD/AD = a/a√3 = 1/√3.

cosec 30° = 1/sin 30° = 2

sec 30° = 1/cos 30°= 2/√3

cot 30° = 1/tan 30° = √3

Similarly, the trigonometric ratios of 60° are:

Sin 60° = a√3/(2a) = √3/2.
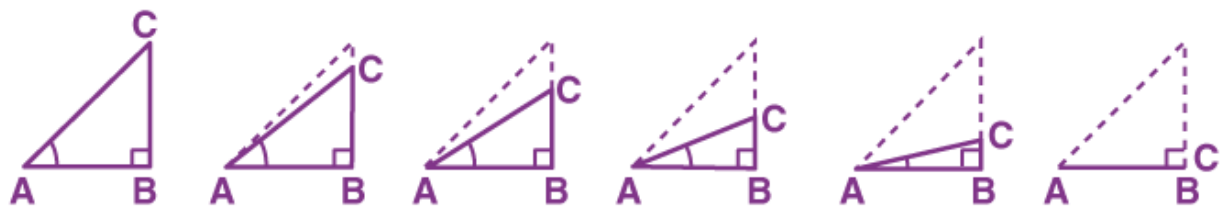
cos 60° = 1/2

tan 60° = √3

cosec 60° = 2/√3

sec 60° = 2

cot 60° = 1/√3

Trigonometric Ratios of 0° and 90°

Consider the same triangle ABC which is right-angled at B. Let us see what happens when the trigonometric ratio of angle A is made smaller and smaller till it becomes zero.

Trigonometric ratio of 0 degree

It is observed that ∠A is very close to 0°, BC also get close to 0. Hence, Sin A = BC/AC is close to 0.

Similarly, if ∠A is very close to 0°, AC is the same as AB, then Cos A = AB/AC = 1

Hence, we can define: **sin 0° = 0 and cos 0° = 1**

With these two observations, we can derive the other ratios of 0°.

tan 0° = sin 0°/cos 0° = 0/1 = 0

cosec 0° = 1/sin 0° = Not defined

sec 0° = 1/cos 0° = 1/1 = 1

cot 0° = 1/tan = 1/0° = Not defined

Similarly, consider the triangle ABC which is right-angled at B. What happens when the trigonometric ratio of angle A is made larger and larger till it becomes 90°.

Trigonometric ratio of 90 degrees
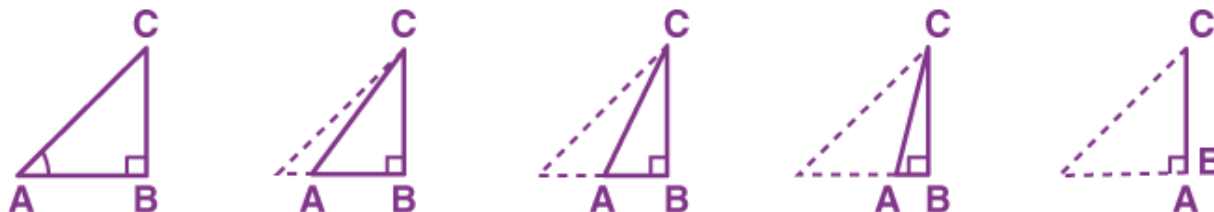
It is observed that if ∠C is very close to 0°, ∠A is very close to 90°, and AC = BC, so sin A is very close to 1. Similarly, when ∠A is very close to 90°, ∠C is very close to 0°, and AB =0, so cos A is very close to 0.

Hence, we can define: **sin 90° = 1 and cos 90° = 0.**

Therefore,

tan 90° = sin 90°/cos 90° = 1/0 = Not defined

cosec 90° = 1/sin 90° = 1/1 = 1

sec 90° = 1/cos 90° = 1/0 = Not defined

cot 90° = 1/tan 90° = 1/ Not defined (∞) = 0.

Trigonometric Ratios of Some Specific Angles Examples

Example 1:

Determine the value of A and B, if sin (A-B) = ½, cos(A+B) = ½, and 0° < A + B ≤ 90°, where A>B.

Solution:

Given that: sin (A-B) = ½

cos(A+B) = ½, and 0° < A + B ≤ 90°, where A>B.

sin (A- B) = ½

Therefore, A-B = Sin-1(½) = 30°….(1)

cos (A+B) = ½

A+B = cos-1(½)

A+B = 60°...(2)

On solving the equation (1) and (2), we get

A = 45° and B = 15°.

Therefore, the values of A and B are 45° and 15°, respectively.

Example 2:

Evaluate the expression sin 60° cos 30° + sin 30° cos 60°.

Solution:

Given Expression: sin 60° cos 30° + sin 30° cos 60°

sin 30° = 1/2

cos 30°= √3/2

sin 60° = √3/2

cos 60°= 1/2

Now, substitute the values of sin 30°, cos 30°, sin 60°, cos 60°,

sin 60° cos 30° + sin 30° cos 60° = (√3/2)(√3/2) + (½)(½)

$$= (¾) + (¼)$$

$$= (3+1)/4$$

$$= 4/4 = 1$$

Therefore, sin 60° cos 30° + sin 30° cos 60° = 1.

Practice Problems

Solve the following problems:

1. Find the value of A and B, if tan (A+B) = √3, and tan (A- B) = 1/√3, and 0°< A + B ≤ 90°, where A > B.

2. Evaluate the expression 2 tan2 45° + cos2 30° − sin2 60°.

3. Is sin θ = cos θ for all values of θ. Justify your answer.

N. B; REVISION QUESTIONS FOR THE TERM IS STUDY PAST QUESTIONS FROM ERC AND BECE COVERING EVERYTHING THAT HAS BEEN TAUGHT FROM JSS1 FIRST TERM TILL DATE FOR BOTH OBJECTIVES AND THEORY.