SURDS

Before we understand vividly what surds are we need to first understand the various types of Number system we have.

Types of Number system

1. Natural numbers: These are set of counting numbers e.g $1, 2, 3, 4, \ldots$. They are used to denote positions such as 1st , 2nd, 3rd, 4th etc. The Natural Number system is denoted by the letter $\mathbb{N}$.
2. Whole numbers: These are set of numbers including all natural numbers and zero (0). The set of whole numbers are denoted by the letter $\mathbb{W}$.
3. Integers : These are sets of negative and positive whole numbers including zero (0). e.g … -3, -2, -1,0, 1, 2,3, … etc. the set of integers are denoted by the letter $\mathbb{Z}$
4. Rational numbers : These are sets of numbers that can be represented in the form $\frac{a}{b}$ where a, b are both integers and $b \neq 0$. E.g. $\frac{5}{1}, \frac{1}{2}, \frac{10}{2}, \frac{2}{1}$.... etc. The set of rational numbers are denoted by the letter $\mathbb{Q}$.

   Furthermore, the set of Rational numbers that can terminate after some term, and if they do not terminate, they will have repeated block.
   E.g. $\frac{1}{2} = 0.5$

   $\frac{1}{4} == 0.25$

5. Irrational numbers: are set of numbers that do not truncate/ terminate and do not have repeated block i.e the decimal digits continues without recurring.
   E.g. $\pi = 3.141592 \ldots$
   $\sqrt{3} = 1.732050 \ldots$

The set of Irrational number are denoted by letter$\mathbb{I}\mathbb{Q}$.

6. <u>Real Number System</u>: Real numbers are set of all possible numbers. It is made up of both the rational and Irrational number system. It is denoted by the $\mathbb{R}$.
7. <u>Complex number system</u>: are sets of numbers which are usually of the form $a + ib$ where a and b are real numbers and $i = \sqrt{-1}$ is an imaginary number. It is denoted by $\mathbb{C}$.

<u>Definition</u> : Surds are Irrational numbers which are in the form of roots. E.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{11},$ etc. are all surds but $\sqrt{16} = 4, \sqrt{9} = 3$, etc are not surds – because they can be simplified to obtain rational numbers.

<u>Definition:</u> Two or more surds are said to be **LIKE SURDS** If the numbers under the square root sign are the same, for example $\sqrt{5}, 6\sqrt{5}, 12\sqrt{5}$ are like surds.

<u>RULES FOR SIMPLIFYING SURDS</u>

1. $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$
2. $\sqrt{m+n} \neq \sqrt{m} + \sqrt{n}$
3. $\sqrt{m-n} \neq \sqrt{m} - \sqrt{n}$
4. $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$
5. $a\sqrt{m} = \sqrt{a^2.m}$
6. $a\sqrt[3]{m} = \sqrt[3]{a^3.m}$

SIMPLIFICATION OF SURDS

To simplify a surds, where possible (because some surds cannot be further simplified), we express the number under the square root sign as a product of two factors, one of which is a perfect square and then simplify the surd by taking the square-root of that which is a perfect square.

Note: A perfect square is a number such that when we take the square-root of it, it will give us a whole number. E.g. 4 is a perfect because $\sqrt{4} = 2$, 16 is a perfect square because $\sqrt{16} = 4$.

Example 1

Simplify the following

1. $\sqrt{45}$
2. $\sqrt{32}$
3. $\sqrt{x\,y^2}$
4. $\sqrt{50}$

Solution

1. $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3 \times \sqrt{5} = 3\sqrt{5}$
2. $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4 \times \sqrt{2} = 4\sqrt{2}$
3. $\sqrt{x\,y^2} = \sqrt{x} \times \sqrt{y^2} = \sqrt{x} \times y = y\sqrt{x}.$
4. $2\sqrt{50} = 2\sqrt{25 \times 2} = 2 \times \sqrt{25} \times \sqrt{2} = 2 \times 5 \times \sqrt{2} = 10\sqrt{2}$

Example 2

Express each of the following as the square root of a single number

1. $2\sqrt{3}$
2. $3\sqrt{10}$
3. $5\sqrt{7}$
4. $2\sqrt{11}$

Solution

Recall the rule: $a\sqrt{m} \;=\; \sqrt{a^2 \times m}$

1. $2\sqrt{3} \;= \sqrt{2^2 \times 3} \;= \sqrt{4 \times 3} \;= \sqrt{12}$
2. $3\sqrt{10} = \sqrt{3^2 \;\times 10} = \;\sqrt{9 \times 10} = \sqrt{90}$
3. $5\sqrt{7} \;= \;\sqrt{5^2 \times 7} \;= \sqrt{25 \times 7} \;= \;\sqrt{175}$
4. $2\sqrt{11} = \sqrt{2^2 \;\times 11} = \sqrt{4 \times 11} = \sqrt{44}$

OPERATIONS WITH SURDS

In mathematics when we talk about operations, we are not referring to surgical operation but we are simply referring to the concept of Addition, Subtraction, Multiplication, Division etc. which are summarized in the word BODMAS and PEDMAS.

Bracket

Of

Division

Multiplication

Addition

Subtraction.

ADDITION AND SUBTRACTION OF SURDS

Two or more surds can be added together or subtracted from one another if and only if they are **LIKE** surds.

Rule For Addition And Subtraction Of Surds

1) $a\sqrt{b} + \; c\sqrt{b} = (a + c)\sqrt{b}$

This is called addition law of surds

2) $a\sqrt{b} - \; c\sqrt{b} = (a - c)\sqrt{b}$

This is called subtraction law of surd.

NOTE: Before addition or subtraction, the surds should first be simplified, if possible.

Example 3

Simplify the following

a) $5\sqrt{3} + 6\sqrt{3}$
b) $7\sqrt{2} - 9\sqrt{2}$
c) $3\sqrt{8} + \sqrt{50}$
d) $2\sqrt{27} + \sqrt{75} - 5\sqrt{12}$
e) $6 + \sqrt{27} + \sqrt{75}$

Solution

a) $5\sqrt{3} + 6\sqrt{3} = (5+6)\sqrt{3} = 11\sqrt{3}$
b) $7\sqrt{2} - 9\sqrt{2} = (7-9)\sqrt{2} = -2\sqrt{2}$

c) $3\sqrt{8} + \sqrt{50}$; Since by mere looking, they are not like surds, so we need to first simplify to check if they are like surds.

So; $3\sqrt{8} + \sqrt{50}$

$$= 3\sqrt{4 \times 2} + \sqrt{25 \times 2}$$
$$= 3 \times \sqrt{4} \times \sqrt{2} + \sqrt{25} \times \sqrt{2}$$
$$= 3 \times 2 \times \sqrt{2} + 5 \times \sqrt{2}$$
$$= 6\sqrt{2} + 5\sqrt{2}$$
$$= (6+5)\sqrt{2}$$
$$= 11\sqrt{2}$$

d) $2\sqrt{27} \ + \ \sqrt{75} - 5\sqrt{12}$

Since by mere looking, they are not surds, so we need to first simplify to check if they are like surds and then proceed.

$2\sqrt{27} \ + \ \sqrt{75} - 5\sqrt{12}$
$= 2\sqrt{9 \times 3} + \sqrt{25 \times 3} - 5\sqrt{4 \times 3}$
$= 2 \times \sqrt{9} \times \sqrt{3} \ + \ \sqrt{25} \times \sqrt{3} - 5 \times \sqrt{4} \times \sqrt{3}$
$= 2 \times 3 \times \sqrt{3} \ + \ 5 \times \sqrt{3} - 5 \times 2 \times \sqrt{3}$
$= 6\sqrt{3} + 5\sqrt{3} - 10\sqrt{3}$
$= (6 + 5 - 10)\sqrt{3}$
$= 1\sqrt{3}$
$= \sqrt{3}$.

e) $6 + \sqrt{27} + \sqrt{75}$
First simplify, we have

$6 + \sqrt{9 \times 3} + \sqrt{25 \times 3}$

$= 6 + \sqrt{9} \times \sqrt{3} + \sqrt{25} \times \sqrt{3}$

$= 6 + 3\sqrt{3} + 5\sqrt{3}$

$= 6 + 8\sqrt{3}$; Since they are not like surds.

MULTIPLICATION OF SURDS.

When two or more surds are multiplied together, they should first be simplified, if possible. Then multiply the whole numbers with whole numbers and surds with surds

That is; $a\sqrt{b} \times c\sqrt{d} = (a \times c)\sqrt{bd}$

This is called the multiplication law of surds.

Example 4

Simplify the following:

a) $\sqrt{27} \times \sqrt{50}$
b) $3\sqrt{12} \times 2\sqrt{60} \times 3\sqrt{45}$
c) $(3\sqrt{5})^2$
d) $\sqrt{5} \times \sqrt{10}$

Solution

a) $\sqrt{27} \times \sqrt{50}$

$= \sqrt{9 \times 3} \times \sqrt{25 \times 2}$

$= 3\sqrt{3} \times 5\sqrt{2}$

$= (3 \times 5) \times \sqrt{3} \times \sqrt{2}$

$= 15\sqrt{6}$

b) $3\sqrt{12} \times 2\sqrt{60} \times 3\sqrt{45}$

$= 3\sqrt{4 \times 3} \times 2\sqrt{4 \times 15} \times 3\sqrt{9 \times 5}$

$= 3 \times 2\sqrt{3} \times 2 \times 2\sqrt{15} \times 3 \times 3\sqrt{5}$

$= (6 \times 4 \times 9) \times \sqrt{3} \times \sqrt{15} \times \sqrt{15}$

$= 72\sqrt{225}$

$= 72 \times 15$

$= 1080$

c) $(3\sqrt{5})^2 = 3\sqrt{5} \times 3\sqrt{5}$

$= (3 \times 3) \times \sqrt{5} \times \sqrt{5}$

$= 9 \times \sqrt{25}$

$= 9 \times 5$

$= 45$

Alternatively;

$\left(3\sqrt{5}\right)^2 = 3^2 \times \left(\sqrt{5}\right)^2$

$= 9 \times \sqrt{5} \times \sqrt{5}$

$= 9 \times \sqrt{25}$

$= 9 \times 5$

$= 45$

d) $\sqrt{5} \times \sqrt{10}$; Since it cannot be further broken, we have;

$= \sqrt{50}$

$= \sqrt{25} \times 2$

$= 5\sqrt{2}$

DIVISION OF SURDS

When one surd divides another, they should first be simplified, if possible. Then divide whole number with whole number and surd with surd.

That is;

$$\frac{a\sqrt{b}}{c\sqrt{d}} = \frac{a}{c} \cdot \frac{\sqrt{b}}{\sqrt{d}} = \frac{a}{c}\sqrt{\frac{b}{d}}$$

This is called the division law surds.

Example

Evaluate the following

a) $\frac{\sqrt{24}}{\sqrt{50}}$

b) $\frac{\sqrt{18}}{\sqrt{2}}$

c) $\frac{\sqrt{5}}{\sqrt{2}}$

d) $\frac{\sqrt{60}\times\sqrt{180}}{\sqrt{75}}$

Solution

$$\frac{\sqrt{24}}{\sqrt{50}} = \frac{\sqrt{4\times 6}}{\sqrt{25\times 2}} = \frac{2\sqrt{6}}{5\sqrt{2}}$$

$$= {}^{2}/_{5}\sqrt{{}^{6}/_{2}}$$

$$= \frac{2}{5}.\sqrt{3} \ = 2\sqrt{3}$$

5

$\frac{\sqrt{18}}{\sqrt{2}} = \frac{\sqrt{9\times 2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{3}{1}\sqrt{\frac{2}{2}}$ $=3\sqrt{1} = 3$

$\frac{\sqrt{5}}{\sqrt{2}} = \sqrt{{}^{5}/_{2} = }\ \sqrt{2}\,\frac{1}{2}$ or $= \sqrt{2.5}$

$$\frac{\sqrt{60}\ \times\sqrt{180}}{\sqrt{75}}$$

$$= \frac{\sqrt{4\times 15}\ \times\ \sqrt{9\times 4\ \times 5}}{\sqrt{75}}$$

$$\frac{2\sqrt{15}\ \ \times 3\times 2\times\sqrt{5}}{\sqrt{75}}$$

$$= \frac{2\sqrt{15}\ \times 6\sqrt{5}}{\sqrt{75}}$$

$$= \frac{(2\times 6\sqrt{15\ \times 5}}{\sqrt{75}}$$

$$= \frac{12\sqrt{75}}{75}$$

$$= \frac{12.}{1}\sqrt{\frac{75}{75}}$$

$$= 12.\sqrt{1}$$

$$= 12$$

Alternatively: $\frac{\sqrt{60} \times \sqrt{180}}{\sqrt{75}}$

$$\frac{\sqrt{10800}}{\sqrt{75}} = \sqrt{\frac{10800}{75}}$$

$$\sqrt{144} = 12$$

SURDS RATIONALIZATION

Rationalization of surds has to do with the process of removing surds from the denominator of a fraction. Hence to rationalize means to make the surds in the denominator to become a rational number. To do this, we multiply the numerator and denominator of the fraction by surds that will make the denominator rational.

N\B: To be rational means to be means to be reasonable (ie to make sense).

Recall: from our knowledge of equivalent fraction, we know that

$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} \frac{a \times \sqrt{b}}{\sqrt{b} \times \sqrt{b}} \quad = \frac{a\sqrt{b}}{\sqrt{b^2}}$

$$= \frac{a\sqrt{b}}{b}$$

Examples

Rationalize the denominators the following:

1 $\frac{2}{\sqrt{2}}$

2 $\frac{10}{\sqrt{5}}$

3 $\frac{4}{\sqrt{8}}$

4 $\sqrt{\frac{16}{7}}$

5 $\frac{5\sqrt{7\ \times\ 2\sqrt{3}}}{\sqrt{45}\ \ \times\ \ \sqrt{21}}$

<u>Solution</u>

1 $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}} = \frac{3\sqrt{2}}{2}$

2 $\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{10\times\sqrt{5}}{\sqrt{5\times\sqrt{5}}} = \frac{10\sqrt{5}}{\sqrt{25}}$

$$= \frac{10\sqrt{5}}{5} \quad 2\sqrt{5}$$

3 $\frac{4}{\sqrt{8}} = \frac{4}{\sqrt{4\times2}} = \frac{4}{2\sqrt{2}} = \frac{4}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{4\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} = \frac{4\sqrt{2}}{2\times\sqrt{4}} = \frac{4\sqrt{2}}{2\times2}$$

$$= \frac{4\sqrt{2}}{4} = \sqrt{2}$$

4 $\sqrt{\frac{16}{7}} = \frac{\sqrt{16}}{\sqrt{7}} = \frac{4}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$$= \frac{4\times\sqrt{7}}{\sqrt{7}\ \times\sqrt{7}} = \frac{4\sqrt{7}}{\sqrt{49}} = \frac{4\sqrt{7}}{7}$$

$$\frac{5\sqrt{7\times2\sqrt{3}}}{\sqrt{45}\ \times\sqrt{21}} = \frac{5\sqrt{7}\ \times2\sqrt{3}}{\sqrt{9\times5}\ \times\ \sqrt{210}}$$

$$= \frac{(5 \times 2) \times \sqrt{7} \times \sqrt{3}}{3\sqrt{5} \times \sqrt{21}}$$

$$= \frac{10 \times \sqrt{21}}{3\sqrt{5} \times \sqrt{21}} = \frac{10}{3\sqrt{5}}$$

Rationalizing the denominator, we have;

$$\frac{10}{3\sqrt{5}} = \frac{10}{3\sqrt{5}} \times \frac{\sqrt{5}}{5} = \frac{10\sqrt{5}}{3 \times \sqrt{5} \times \sqrt{5}}$$

$$= \frac{10\sqrt{5}}{3 \times \sqrt{25}} = \frac{10\sqrt{5}}{3 \times 5}$$

$$= \frac{10\sqrt{5}}{15} \qquad = \frac{2\sqrt{5}}{3}$$

NOTE; $\sqrt{a} \times \sqrt{a} = a$

Proof:

$$\sqrt{a} \times \sqrt{a} = \sqrt{a^2}$$

$$= a.$$

Hence; $\sqrt{3} \times \sqrt{3} = \sqrt{9} = \sqrt{9} = 3$

$$\sqrt{4} \times \sqrt{4} = \sqrt{16} = 4$$

$$\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$$

WHY DO WE NEED TO RATIONALIZE THE DENONATOR OF A SURD

Consider the Example below

Example

Given that $\sqrt{3} = 1.732$, evaluate $\frac{2}{\sqrt{3}}$to 2 decimal places.

Solution

By rationalizing the denominator, we have

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$= \frac{2 \times 1.732}{3}$$

$$= \frac{3.464}{3}$$

$$= 1.154 \ldots.$$

$= 1.15$to2 decimal places without rationalizing we have

$$\frac{2}{\sqrt{3}} = \frac{2}{1.732} = 1.154 \ldots$$

$$= 1.15 \text{ to 2 d.p}$$

This last step involves division by 1.1732 which would normally require the use of a calculator unlike the first method which is less cumbersome.

Hence, the process of rationalizing the denominator is more advantages.

SURDS IN BRACKETS

NOTE: When multiplying surds in brackets, we use algebraic expansion:

$$(a + b)(c + d)$$

$$= a(c + d) + b\,(c + d)$$

$$= ac + ad + bc + bd.$$

Example

Simplify the following :

a) $(3\sqrt{5} + 2)\ (\sqrt{5} + 3)$
b) $(4\sqrt{3}\ + \sqrt{2}\ )\ (4\sqrt{3} - \sqrt{2)}$
c) $(3\sqrt{2} - 6)^2$
d) $(5+3\sqrt{2})\ (5 - 3\sqrt{2})$

Solution

a) $\left(3\sqrt{5} + 2\right)(5 + 3)$
$= 3\sqrt{5}\left(\sqrt{5} + 3\right) + 2(\sqrt{5} + 3)$
$= \left(3\sqrt{5}\ \times \sqrt{5}\right) + \left(3\sqrt{5}\ \times 3\right) + 2\sqrt{5} + (2 \times 3)$
$= 3\sqrt{25} +\ 9\sqrt{5} + 2\sqrt{5} + 6$
$= 3(5) +\ 9\sqrt{5} + 2\sqrt{5} + 6$
$= 15 + 9\sqrt{5} + 2\sqrt{5} + 6$
Collecting like terms
$= (15 + 6) +\ \left(9\sqrt{5}\ \ + 2\ \sqrt{5}\right)$

$= 21 + 11\sqrt{5}$

b) $(4\sqrt{3} + \sqrt{3} - \sqrt{2)}$
$= 4\sqrt{3}(4\sqrt{3} - \sqrt{2)} + \sqrt{2}\ (\ 4\sqrt{3} - \sqrt{2)}$
$= (4\sqrt{3}\ \times 4\sqrt{3)} - (4\sqrt{3} \times \sqrt{2)} +\ \left(\sqrt{2} \times 4\sqrt{3}\right) - (\sqrt{2} \times \sqrt{2})$
$= (16 \times 3) -\ 4\sqrt{6} +\ 4\sqrt{6} - 2$
$= 48 - 4\sqrt{6} + 4\sqrt{6} - 2$

Collecting like terms

$= (48 - 2) - 4\sqrt{6} + 4\sqrt{6}$

$= 48 - 2$

$= 46$

c) $(3\sqrt{2} - 6)^2$

$= 3\sqrt{2} - 6)\ (3\sqrt{2} - 6)$

$= 3\sqrt{2}(3\sqrt{2} - 6) - 6(3\sqrt{2} - 6)$

$= (3\sqrt{2}\ \times 3\sqrt{2)} - (3\sqrt{2} \times 6) - (6 \times 3\sqrt{2}) + (6 \times 6)$

$= (9 \times 2) - 18\sqrt{2} - 18\sqrt{2} + \ 36$

Collecting like terms

$= (18 + 36) - \ 18\sqrt{2} - 18\sqrt{2}$

$= 54 - 36\ \sqrt{2}$

d) $(5 + 3\sqrt{2})(5 - 3\sqrt{2})$

$= 5(5 - 3\sqrt{2}) + 3\sqrt{2}(5 - 3\sqrt{2})$

$= (5 \times 5) - (5 \times 3\sqrt{2}) + (3\sqrt{2}\ \times 5) - (3\sqrt{2}\ \times 3\sqrt{2})$

$= 25 - 15\sqrt{2} + \ 15\sqrt{2} - (9 \times 2)$

Collecting like terms

$$= (25 - 18) - 15\sqrt{2} + 15\sqrt{2}$$

$= 7$

CONJUGATE SURDS

Two surds are said to be conjugate of each other if their product gives rise to a rational number.

From our knowledge of difference of two squares:

$$a^2 - b^2 = (a + b)\ (a - b)$$

Proof

$(a + b)(a - b) = a(a - b) + b(a - b)$

$$= a^2 - ab + ab - b^2$$

$$= a^2 - b^2$$

Hence $(a + b)(a - b) = a^2 - b^2$

Similarly;

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b)}$

$$= \sqrt{a}\,(\sqrt{a} - \sqrt{b)} + \sqrt{b}(\sqrt{a} - \sqrt{b})$$

$= (\sqrt{a} \times \sqrt{a}) - (\sqrt{a} \times \sqrt{b}) + (\sqrt{b} - \sqrt{a}) - (\sqrt{b} \times \sqrt{b})$

$= (a - \sqrt{ab} + \sqrt{ab} - b$

$a - b$

N/B: $\sqrt{a} + \sqrt{b}$ *and* $\sqrt{a} - \sqrt{b}$

Are not rational, but their product $a - b$ is rational.

Hence $\sqrt{a} + \sqrt{ab} - b$ and $\sqrt{a} - \sqrt{b}$ are conjugate of each of each other.

Also; $(a + \sqrt{b})(a - \sqrt{b})$

$= a(a - \sqrt{b}) + \sqrt{b}\,(a - \sqrt{b)}$

$= a^2 - a\sqrt{b} + a\sqrt{b} + (\sqrt{b} \times \sqrt{b})$

$= a^2 + b$ which is a rational number.

Hence; $a + \sqrt{b}$ *and* $a - \sqrt{b}$ are conjugate of each other.

Example

Write down the conjugate of the following a$\sqrt{3} - 5\sqrt{2}$

b$3\sqrt{5} + 3\sqrt{3}$

c$5\sqrt{5} - 4\sqrt{2}$

d $8 + \sqrt{5}$

e$-2\sqrt{7} - \sqrt{5}$

f $5 + 3\sqrt{2}$

Solution

a) The conjugate of $\sqrt{3} - 5\sqrt{2}$ is $\sqrt{3} + 5\sqrt{2}$
b) The conjugate of $3\sqrt{5} + 3\sqrt{3}$ is $3\sqrt{5}\ 3\sqrt{3}$
c) The conjugate of $5\sqrt{5} - 4\sqrt{2}$ is $5\sqrt{5} + 4\sqrt{2}$
d) The conjugate of $8 + \sqrt{5}$ is $8 - \sqrt{5}$
e) The conjugate of $-2\sqrt{7} - \sqrt{5}$ is $-2\sqrt{7} + \sqrt{5}$
f) The conjugate of $5 + 3\sqrt{2}$ is $5 - 3\sqrt{2}$.

N/B In conjugate, we simply change the signs connecting the two terms.

The concepts of conjugation are very useful in rationalizing denominators with binomial surds.

Since a binomial surds means an expression with two terms. Hence a binomial surd is an expression that either both are surds or one is a surd

e.g are $a + \sqrt{b}$ or

$$\sqrt{a} + \sqrt{b}.$$

FURTHERCONCEPTON SURDS.

Pre- requisite: Student should be familiar with how to solve simultaneous quadratic equations before studying this further concept on surds.

EQUITY OF SURDS

Two binomial surds $a+\sqrt{b}\ \ and\ c+\sqrt{d}\ are\ said\ to\ be\ equal\ if\ and\ \ only\ if\ a=c\ and\ b=d.$

$$That\ is\ a+\sqrt{b}=c+\sqrt{d}$$

$$\Longrightarrow \quad a=c\ and\ \ b=d.$$

Example

If $x+\sqrt{2}=10+\sqrt{2}.\ find\ the\ vaue\ of\ x.$

Solution

Since $x+\sqrt{2}=10+\sqrt{2}$

From the concept of equality

$\Longrightarrow \quad x=10.$

FINDING THE SQUARE OF ROOT OF A SURD NUMBER.

N/B. The concept of equality of two surds enables us to determine the square roots of a surdic number.

Theorem

The square root of a surdic number, $a+\sqrt{b}\ \ or\ a-\sqrt{b}\ \ is\ always\ of\ the\ form$.

$$\sqrt{a+\sqrt{b}}\ =\sqrt{x}+\sqrt{y}$$

And

$$\sqrt{a-\sqrt{b}} = \sqrt{x} - \sqrt{y}$$

Example:1

Find $\sqrt{5+2\sqrt{6}}$

Solution.

Let $\sqrt{5+2\sqrt{6}} = \sqrt{x} + \sqrt{y}$

Squaring both sides, we obtain

$$\Longrightarrow (\sqrt{5+2\sqrt{6})^2} = (\sqrt{x} + \sqrt{y})^2$$

Expanding we have:

$\Longrightarrow\ 5 + 2\sqrt{6} = x + 2\sqrt{xy} + y$

$\Longrightarrow\ 5 + 2\sqrt{6} = x + y + 2\sqrt{xy}$

From the concept of equality of surds:

$\Longrightarrow 5 = x + yand6 = xy$.

$\Longrightarrow x + y = 5 \longrightarrow (1)$

$\Longrightarrow xy = 5 \longrightarrow \quad (2)$

Solving Simultaneously;

From equation (1): $x = 5 - y \longrightarrow (3)$

Substituting into equation (2) we have :

$(5 - y)y = 6$

$\Longrightarrow 5y - y^2 = 6$

Rearranging, we have ;

$$y^2 - 5y + 6 = 0$$

Solving the quadratic equation, we have :

$y^2 - 3y - 2y + 6 = 0$

$\Longrightarrow y(y-3) - 2(y-3) = 0$

$\Longrightarrow (y-2)(y-3) = 0$

$\Longrightarrow either\ y - 2 = 0\ or\ y - 3 = 0$

$\Longrightarrow y = 2\ or\ y = 3.$

$To\ find\ x, we\ substitute\ the\ values\ of\ y\ into\ any\ of\ the\ above\ equations\ equation\ (3)$

$for\ instance$;

$x = 5 - y$

$\Longrightarrow when\ y = 2;\ \ x = 5 - 2 = 3$

$Also\ when\ y = 3, x = 5 - 3 = 2$

Thus, $x = 3\ when\ y = 2$

$and\ x = 2\ when\ y = 3$

Hence since the square root of

$5 + 2\sqrt{6} = \sqrt{x} + \sqrt{y}$

$\Longrightarrow \sqrt{5 + 2\sqrt{2}} = \sqrt{3} + \sqrt{2}$or

$$\sqrt{5 + 2\sqrt{6}}\sqrt{2} + \sqrt{3}$$

N/B: Since $\sqrt{3} + \sqrt{2} = \sqrt{2} + \sqrt{3}$ $\{commutative$

$property\ of\ numbers\}$ So any of the is true.

The above solution can be verified using a scientific calculator;/Tables

That is;

$$\sqrt{5 + 2\sqrt{6}} \quad = \quad \sqrt{3} + \sqrt{2}$$

$\Longrightarrow \sqrt{5 + 4.9} \quad = 1.732 + 1.414$

$= \sqrt{9.9} \qquad = 3.146$

$= 3.146 \qquad = 3.146$

Hence $\sqrt{5 + 2\sqrt{6}} = \sqrt{3} + \sqrt{2}$.

Example (2).

Find the square root of $7 + 2\sqrt{10}$.

<u>Solution</u>

Let $\sqrt{7 + 2\sqrt{10}} \quad = \quad \sqrt{x} + \sqrt{y}$

Squaring both sides, we have:

$(\sqrt{7 + 2\sqrt{10}})^2 \quad = \quad (\sqrt{x} + \sqrt{y})^2$

$\Longleftarrow 7 + 2\sqrt{10} \quad = \quad (\sqrt{x} + \sqrt{y})^2$

Expanding we have:

$7 + 2\sqrt{10} \quad = \quad x + 2\sqrt{xy} + y$

$\Longrightarrow 7 + 2\sqrt{10} \quad = \quad x + y + 2\sqrt{xy}$.

From the concept of equality of surds;

$\Longrightarrow 7 = x + y \;\; and \; xy = 10$

$\Longrightarrow x + y = 7,,,,,,,,,,,,,,,,(1)$

$$xy = 10 ,,,,,,,,,,,,,,,,,, (2)$$

Solving Simultaneously;

From equation (1)$x = 7 - y \dots \dots \dots \dots \dots .. (3)$

Substituting into equation (2), we have

$(7 - y)y = 10$

$$\Longrightarrow 7y - y^2 = 10$$

Rearranging we have.

$$y^2 - 7y + 10 = 0.$$

Solving the quadratic equation, we have

$y^2 - 7y + 10 = 0$

$\Longrightarrow y^2 - 5y - 2y + 10 = 0$

$\Longrightarrow y(y - 2) - 2(y - 5) = 0$

$\Longrightarrow (y - 2)(y - 5) = 0$

$either\ y - 2 = 0\ or y - 5 = 0$

$\Longrightarrow y = 2\ or\ y = 5$

To find x, we Substitute the values of y into any of the above equations

Using equations; (3) $\quad x = 7 - y$

When $y = 2, \Longrightarrow \quad when\ y = 5.$

Hence, since the square – root of 5.

$7 + 2\sqrt{10} = \sqrt{x} \quad + \quad \sqrt{y}.$

$\Longrightarrow \sqrt{7 + 2\sqrt{10}} = \sqrt{5} + \sqrt{2}$or

$$\sqrt{7 + 2\sqrt{10}} = \ +\sqrt{2} + \sqrt{5}$$

Since by commutativity: $\sqrt{5} + \ \sqrt{2} = \ \sqrt{2} + \sqrt{5}$

$\Longrightarrow$ *any of the above solution is true.*

N/B: The above solution can be verified using a scientific calculator\tables.

Example (3)

Find the square- root of $14 - 4\sqrt{6}$.

Solution

Let $\sqrt{14 - 4\sqrt{6}} = \sqrt{x} - \ \sqrt{y}$.

Squaring both sides we obtain.

$(\sqrt{14 - 4\sqrt{6}})^2 = (\sqrt{x} - \sqrt{y})^2$

$$\Longrightarrow 14 - 4\sqrt{6} \ = x - 2\sqrt{xy} + \ y$$

$\Longrightarrow 14 - 4\sqrt{6} = x + y - 2\sqrt{xy}$

From the concept of equality of surds.

$\Longrightarrow 14 = x + y \text{ and } 4\sqrt{6} = 2\sqrt{xy}$

$\Longrightarrow x + y = 14$ …………………(1)

And $2\sqrt{xy} \ \ = \ 4\sqrt{6} \ldots \ldots \ldots \ldots \ldots ..$

From equation ( ) $\Longrightarrow \ \sqrt{xy} \ = 2\sqrt{6}$

$$\Longrightarrow \ \sqrt{xy} = \ \sqrt{2^2 x 6}$$

$$\Longrightarrow \sqrt{xy} = \sqrt{24}$$

$$\Longrightarrow xy = 24 \quad \dots \dots .(2)$$

Hence; $x + y = 14 \dots \dots \dots (1)$

And $xy = \quad 24 \dots \dots \dots \dots \dots (2)$

From equation (1); $\qquad x = 14 - y \dots \dots \dots \dots .(3)$

Substituting into equation (2) we have:

$(14 - y)y = 24$

$\Longrightarrow 14y - y^2 = 24$

Rearranging we have;

$y^2 - 14y + 24 = 0$

Solving the quadratic equation, we have;

$y^2 - 12y - 2y + 24 = 0$

$\Longrightarrow y(y - 12) - 2(y - 12) = 0$

$\Longrightarrow \quad (y - 2)(y - 12) = 0$

$$either \quad y - 2 = 0 \; or \; y - 12 = 0$$

$\Longrightarrow \quad y = 2 \; or \; y = 12$

To find the values of x, we substitute the values of y into any of the above equations;

Using equation (3):$x = 14 - y$

$when\; y = \; 2; \; \Longrightarrow \; x = 14 - 2 = 12$

$when \quad y = 12, \qquad \Longrightarrow x = 14 - 12 = 2$

Thus $x = 12, when\; y = 2$

And $x = 2\ when\ 12\ , when\ y = 12$.

Hence, since the square- root of

$14 - 4\sqrt{6} = \sqrt{x}\ - \sqrt{y}$

$$\Rightarrow \sqrt{14 - 4\sqrt{6}}\ =\ \sqrt{12} - \sqrt{2}$$

$$\sqrt{14 - 4\sqrt{6}}\ \ =\ \sqrt{2} - \sqrt{12}$$

EQUATION IN IRRATIONAL FORMS.

Recall that: to solve an equation simply means to find the unknown value denoted by a letter.

Here we shall be considering equations in a variable and in surds

(ie irration) form.

An equation of the for m

$a\ \sqrt[n]{bx + c}\ \ = d\ where\ a,\quad b,$
$c\ and\ d\ are\ constant\ numbers, x\ is\ variable\ \ and\ n\ is\ the\ root,$
$is\ called\ a\ RADICAL\ EQUATION\ of\ nth\ order, while\ the\ expression\ under\ the\ root$

$$sign\ is\ called\ the\ RADICAND.$$

$Example\ 2\sqrt{3x + 4} +\ x = 36\ is\ a\ radical\ equation\ of\ the\ second\ order.$

Example

Solve the radical equation:

$$2\sqrt{3x} + 4\ + x = 36$$

$$Solution$$

$$2\sqrt{3x+4} + \ x = 36$$

Isolating the term containing the radical we have:

$2\,\sqrt{3x+4}\ = 36 - x$

$\Longrightarrow \sqrt{2^2(3x+4) = 36} - x$

Squaring both side we have ;

$4(3x+4) = (36-x)^2$

$\Longrightarrow 12x + 16 = (36-x)(36-x)$

Example

Solve the equation

$\sqrt{x+8}\ + \ \sqrt{x+1} = 7$

Solution

$\sqrt{x+8} + \ \sqrt{x+1} = 7$

Squaring both sides of the equation

$\Longrightarrow \quad (\sqrt{x+8}\sqrt{x+1})^2 = 7^2$

$\Longrightarrow (x+8) + 2\,\sqrt{(x+8)(x+1)} = 49$

$\Longrightarrow \quad x + 8 + x + 1 + 2\sqrt{(x+8)(x+1)} = 49$

$\Longrightarrow \ 2x + 9 + 2\,\sqrt{(x+8)(x+1)} = 49$

$Isolating\ the\ radical\ term\ we\ have$:

$2\sqrt{(x+8)(x+1)} \; = 49 - 9 - 2x$

$\Longrightarrow 2\sqrt{(x+8)(x+1)} \; = 40 - 2x$

$\Longrightarrow \sqrt{2^2 \,.\, (x+8)(x+1))} = 40 - 2x.$

$\Longrightarrow \sqrt{4(x+8)(x+1)} = 40 - 2x$

Squaring both sides, we have

$4(x+8)(x+1) = (40-2x)^2$

$\longrightarrow 4(x+8)(x+1) = (40-2x)(40-2x)$

$\Longrightarrow \; 4(x+8)(x+1) = 1600 - 80x - 80 + 4x^2$

$\Longrightarrow 4(x+8)(x+1) = 4x^2 - 160x + 1600$

$\Longrightarrow 4\lfloor x^2 + x + 8x + 8 \rfloor \; = 4x^2 - 160x + 1600$

$$\Longrightarrow 4x^2 + 36x + 32 = 4x^2 - 160x + 1600$$

$\Longrightarrow 4x^2 + 36 + 32 - 4x^2 + 160x + 1600 = 0$

$196x - 1568 = 0$

$\Longrightarrow 196x = 1568$

$x = \dfrac{1568}{196}$

$\Longrightarrow \quad x = 8$

To check

Using the given equation

$\sqrt{x+8} \; + \; \sqrt{x+1} = 7$

$$puting \; x = 8, we \; have$$

$\sqrt{8+8} + \; \sqrt{8+1} = 7$

$\Longrightarrow\ \sqrt{16} + \sqrt{9} = 7$

$$\Longrightarrow \quad 4 + 3 = 7$$

$\Longrightarrow\ \ 7 \ = \ 7$

$Hence\ x = 8\ is\ the\ solution\ or\ the\ root\ of\ the\ equation.$

<u>Example</u>

Solve the equation $\sqrt{(3x+1)} - \sqrt{(x+4)} = 1$

Re-arranging the equation, we have

$\sqrt{(3x+1)} = 1 + \sqrt{(x+4)}$

Squaring both sides we have:

$[\sqrt{(3x+1)}]^2\ [1 + \sqrt{(x+4)}]^2$

$\Longrightarrow 3x + 1\ = \left[1 + \sqrt{(x+4)}\right]\left[1 + \sqrt{(x+4)}\right]$

$\Longrightarrow 3x + 1 = 1 + 2\sqrt{(x+4)} + x + 4$

$\Longrightarrow 3x + 1 = x + 5 + 2\sqrt{(x+4)}$

$\Longrightarrow 3x + 1 - x - x - 5 = 2\sqrt{x+4}$

$\Longrightarrow 2x - 4 = 2\sqrt{x+4}$

$\Longrightarrow 2(x-2) =\ \sqrt{x+4}$

$\Longrightarrow\ \ x - 2 =\ \dfrac{2\sqrt{x+4}}{2}$

$\Longrightarrow\ \ x - 2 = \sqrt{x+4}$

Squaring both sides again;

$\Longrightarrow (x-2)^2 = x+4, expanding\ we\ have;$

$\Longrightarrow \ x^2 - 4x + 4 = x + 4$

$\Longrightarrow \ x^2 - 4x + 4 - x - 4 = 0$

$\Longrightarrow \ x^2 - 5x = 0$

$\Longrightarrow \ x(x-5) = 0$

$either\ x = 0\ \ or\ x - 5 = 0$

$$\Longrightarrow \ x = 0\ or\ x = 5$$

To check :

Using the given equation:

$\sqrt{(3x+1)} - \sqrt{(x+4)} = 1$

Putting $x = 0$, $we\ have$

$\sqrt{(3(0)+1)} - \sqrt{(0+4)} = 1$

$\Longrightarrow \ \sqrt{1} - \sqrt{4} = 1$

$= 1 - 2 = 1$

$= \ -1 \ \neq 1$

Hence $x = 0\ is\ NOT\ a\ solution$

Also putting $x = 5, then\ we\ have;$

$\sqrt{(3(5)+1)} - \sqrt{(5+4)} = 1$

$\sqrt{16} \ - \sqrt{9} = 1$

$4 - 3 = 1$

Hence ,$x = 5\ is\ a\ solution\ or\ a\ root\ of\ the\ equation.$

N\B: We call $x =$

$0\ (which\ is\ not\ a\ solution) an\ extraneous\ root\ of\ the\ original\ equation.$

RATIONALIZATION OF DENOMINATION WITH BINOMIAL SURDS,

From our knowledge of conjugate surds, if the denominator of a fraction is of the form

$\sqrt{a} + \sqrt{b},\ it\ can\ be\ rationalized\ by\ multipling\ the\ entire\ fraction\ by\ \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

Similarly if the denominator of the fraction is of the form $a + b, it\ can\ be\ rationalized\ by\ multiplying\ the\ entire\ fraction\ by\ \frac{a-\sqrt{b}}{a-\sqrt{b}}$

Example

Simplify the following by rationalizing the denominators

a) $\frac{3+\sqrt{2}}{\sqrt{5}+\sqrt{3}}$
b) $\frac{6}{2\sqrt{2}-1}$
c) $\frac{2}{3\sqrt{5}+4}$
d) $\frac{1+\sqrt{5}}{1-\sqrt{5}}$
e) $\frac{8-3\sqrt{6}}{2\sqrt{3}+3\sqrt{2}}$
f) $\frac{2}{(3\sqrt{5-4})^2}$

To include,

Example of surds are$\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{7}, \sqrt{11},\ etc.$

We shall focus on surds which are square roots of rational numbers e.g $\sqrt{2},\ \sqrt{3}, \sqrt{7},\ etc. These\ surds\ are\ called\ quadratic\ surds.$

N/B: roots of national numbers which are irrational numbers called surds.

Basic forms of a surds:

The surds $\sqrt{p}\ is\ said\ to\ be\ in\ its\ basic\ form\ of\ P\ can\ not\ be\ broken\ further\ into\ two\ factor\ where\ at\ least\ one\ of\ the\ factor\ is\ a\ perfect\ square\ e.g$

$\sqrt{6}, \sqrt{5}, \sqrt{2}, \sqrt{3}\ etc$ are in basic forms because none can be further broken down into two factor where at least one is perfect square.

$But\ \sqrt{20}\ is\ not\ in\ basic\ form\ because\ it\ can\ be\ further\ broken\ into\ \sqrt{4 \times 5}\ where\ 4\ is\ a\ perfect\ square.$

i.e $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

N/B: A perfect square is a number that when you take it square root, it gives us a whole number. e.g 4 is a perfect square because $\sqrt{4} = 2, etc.$

INDICES

LAWS OF INDICES

Indices is the plural for index, it simply means power or exponent.

Index notation is the method of shortening the product of numbers of equal factors or numbers that are the same.

For example: $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$$9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^7$$

Generally, if we have: $a \times a \times a = a^3$, the "$a$" is called the **base** while "3" is the **power** or **index.**

That is: $\boldsymbol{a}^{\boldsymbol{m}}$ --------- ***index***
………………. ***base***

The following laws of indices are true for all values of a, m and n provided $a \neq 0$.

1. **Multiplication law:**
   $\boldsymbol{a^m \times a^n = a^{m+n}}$
   In words, this rule states that: when multiplying powers of the same base, simply **add** the indices.
   Example: a). $8^3 \times 8^4 = 8^{3+4} \quad = 8^7$
   b). $x^{-5} \times x^4 = x^{-5+4} \quad = x^{-1}$
   c). $3^8 \times 3^{-4} = 3^{8+(-4)} = 3^{8-4} \quad = 3^4$
   Note: This rule works for both positive and negative indices.

2. **Division law:**
   $\boldsymbol{a^m \div a^n = a^{m-n}}$
   In words, this rule states that: when dividing powers of the same base, simply **subtract** the second index from the first.
   Example: a). $8^9 \div 8^4 = 8^{9-4} \quad = 8^5$
   b). $x^{-5} \div x^4 = x^{-5-4} \quad = x^{-9}$
   c). $3^8 \div 3^{-4} = 3^{8-(-4)} = 3^{8+4} \quad = 3^{12}$

Note: This rule works for both positive and negative indices.

3. **Zero power law (Zero index):**

   $\boldsymbol{a^0 = 1}$

   In words, this rule states that: when any number (except, 0) is raised to the power of zero, the result is 1.

   Example: a). $8^3 \div 8^3 = 8^{3-3} = 8^0 = 1$

   b). $x^{-5} \times x^5 = x^{-5+5} \quad = x^0 = 1$

   c). $3654000^0 \quad = 1$

4. **Negative power law (Negative Index):**

   $$\boldsymbol{a^{-m} = \frac{1}{a^m}}$$

   In words, this rule states that: when a number is raised to the power of a negative index, then it is equal to the ***reciprocal*** of the number raised to the power of the positive index.

   Example: a). $x^{-3} = \frac{1}{x^3}$

   b). $x^{-5} \times x^3 = x^{-5+3} \quad = x^{-2} = \frac{1}{x^2}$

   c). $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

5. **Power law:**

   $\boldsymbol{(a^m)^n = a^{mn}}$

   In words, this rule states that: when a number raised to a power is raised to another power, simply **multiply** the indices.

   Example: a). $(8^3)^4 = 8^{3\times4} \quad = 8^{12}$

   b). $(x^{-3})^{10} = x^{-3\times10} = x^{-30} = {}^{1}/_{x^{30}}$

   c). $(c^{-3})^{-3} = c^{-3\times-3} = c^9$

   Note: This rule works for both positive and negative indices.

6. **Product power law:**

$(\boldsymbol{ab})^{\boldsymbol{m}} = \boldsymbol{a}^{\boldsymbol{m}}\boldsymbol{b}^{\boldsymbol{m}}$

In words, this rule states that: when the product of different factors is raised to a certain power, then the power is distributed over each factor.

Example: a). $(3xy)^3 = 3^3 \times x^3 \times y^3$

$= 27x^3y^3$

b). $(-2x^6)^4 = (-2)^4 \times (x^6)^4$

$= 16 \times x^{6\times4}$

$= 16x^{24}$

<u>LOGARITHMS</u>

INTRODUCTION: Logarithm simply means "**power**".

Consider the following short forms;

$$10000 = 10^4$$

$$1000 = 10^3$$

$$100 = 10^2$$

$$10 = 10^1$$

$$1 = 10^0$$

Which are true. Picking one of the expressions, say,

$$10000 = 10^4$$

Mathematicians interprets the above to mean that "the **logarithm,** to the base of 10, of 10000 is 4", or for short,

"$\log_{10} 10000 = 4$"

DEFINITION: The **logarithm** of a number (say, N) to the base of another number $(say, a\ )$ written as $\mathbf{log}_{\boldsymbol{a}} \boldsymbol{N}$is the **power** to which $'a'$ is raised to give that number N.

That is,

$$\boldsymbol{If\ a^x = N}$$

$$\boldsymbol{then}, \mathbf{log}_{\boldsymbol{a}} \boldsymbol{N = x}$$

Meaning that, the logarithm (or power) of N to the base $'a'$ is equal to x.

Hence the logarithm of a number, N to the base 10 $(called\ the\ common\ logarithm)$ written as $\log_{10} N$ is the power to which 10 is raised to give that number N.

That is,

$$If\ 10^x = N$$

$$then,\ \log_{10} N = x$$

N/B: the base of any logarithm of a number can **never** be $0\ or\ 1$. this is because $0\ or\ 1$ cannot be raised to any number to give us another number except itself,

$That\ is, 0^2 = 0^3 = \cdots = 0^n = 0\ and\ 1^2 = 1^3 = \cdots = 1^n = 1$, where n is any number.

Examples:

1. $If\ \ 1000 = 10^3\ then\ \Longrightarrow\ \log_{10} 1000 = 3$
2. $If\ \ 100 = 10^2\ then\ \Longrightarrow\ \log_{10} 100 = 2$
3. $If\ \ 10 = 10^1\ then\ \Longrightarrow\ \log_{10} 10 = 1$
4. $If\ \ 1 = 10^0\ then\ \Longrightarrow\ \log_{10} 1 = 0$
5. $If\ \ 8 = 2^3\ then\ \Longrightarrow\ \log_2 8 = 3$
6. $If\ \ 16 = 2^4\ then\ \Longrightarrow\ \log_2 16 = 4$

and so on.

N/B: A good knowledge of standard forms is essential for the understanding of logarithms.

REVISION ON STANDARD FORM

Standard form is a scientific form of expressing numbers in powers of 10.

E.g.

$$1000 = 10^3$$

$$100 = 10^2$$

$$10 = 10^1$$

$$1 = 10^0$$

$$0.1 = 10^{-1}$$

$$0.01 = 10^{-2}$$

$$0.001 = 10^{-3}$$

Definition:A number in the form $A \times 10^n$, where A is a number between 1 and $10 (i.e. 1 \leq A < 10)$ and $'n'$ is an Integer, is said to be in ***standard form.*** For e.g. 3.43×10^2 and 8.4×10^{-4} are numbers in standard form.

N/B: Integers are set of positive and negative whole numbers including zero (0).

Example:

Express the following numbers in standard form;

a. 243**b.** 4123**c.** 0.32**d.** 0.0013

NOTE:

1. In converting an ordinary number to standard form, the rule states that: the decimal point in the ordinary number must be brought to the point in-between the first two whole numbers. The first whole number must be a number greater than 0.

2. Every number or value contains one decimal point. E.g. 234 can be written as 234.00000 to any number of zeros you desire. Also, 100 can be written as 100.00 etc. so in converting to standard form, we move the decimal point to the point in-between the first two whole numbers.

Solution:

a. 243 which can be written as 243.00000

Hence, $243 = 2.43 \times 100$ {since we moved the decimal point twice to the left just to get to the point in-between the first two whole numbers, so we have 10^2 which is 100.}

$\Longrightarrow \quad 243 = 2.43 \times 10^2.$

b. 4123 which can be written as 4123.00

Hence, $4123 = 4.123 \times 1000$ {since we moved the decimal point thrice to the left just to get to the point in-between the first two whole numbers, so we have 10^3 which is 1000.}

$\Longrightarrow \quad 243 = 2.43 \times 10^2.$

c. $0.32 = 3.2 \times 0.1$ {since we moved the decimal point once to the right just to get to the point in-between the first two whole numbers, so we have 10^{-1} which is 0.1}

$\Longrightarrow \quad 0.32 = 3.2 \times 10^{-1}.$

d. $0.0013 = 1.3 \times 0.001$ {since we moved the decimal point thrice to the right just to get to the point in-between the first two whole numbers, so we have 10^{-3} which is 0.001}

$\Longrightarrow \quad 0.0013 = 1.3 \times 10^{-3}.$

LOGARITHMS OF NUMBERS GREATER THAN 1.

Logarithms used in calculations are usually in base 10. They are called **common logarithms.** We simply write $\log_{10} 100$as $\log 100$. Meaning that there is no need to write down the base when using common logarithms. The logarithm of a number to the base 10 can easily be obtained using a **four-figure logarithm table** (which was published about 400 years ago).

The logarithm of a number is in two parts;

- The characteristic (integer or whole number) part
- The mantissa (decimal fraction) part.

For instance, the logarithm of 1000 $(\text{i.e.} \log 1000)$ is

3.0000

$characteristic mantissa$

The characteristic of a logarithm

The characteristic is the integer (or integral) or whole number part of a logarithm.

N/B: If a number is in standard form, it is easy to see what the characteristic of its logarithm is. For example, $\log 743$ has the characteristic $'2'$ because

$743 = 7.43 \times 10^2$

and the characteristic of $\log 5203$ is 3 because

$5203 = 5.203 \times 10^3$.

Example:

State the characteristics of the logarithms of the following numbers

a. 960 b. 46 400 c. 42.6 d. 1234

Solution:

Using the standard form method

a. $960 = 9.60 \times 10^2$ $\Longrightarrow$ the characteristics $= 2$

b. $46\ 400 = 4.64 \times 10^4$ $\Longrightarrow$ the characteristics $= 4$

c. $42.6 = 4.26 \times 10^1$ $\Longrightarrow$ the characteristics $= 1$

d. $1234 = 1.234 \times 10^3$ $\Longrightarrow$ the characteristics $= 3$

N/B: the characteristic of a logarithm can also be determined **by careful observation** of the number of digits before the decimal point. We can notice that for whatever figure whose logarithm is sought, the characteristic is always **one less than** the number of digits in the whole number part of the given figure(or number).

Example:

State the characteristics of the logarithms of the following numbers by careful observation.

a. 960 b. 46 400 c. 42.6 d. 1234

Solution:

By careful observation, we see that;

Numbers	**Characteristics**
a. 960	2
b. 46400	4
c. 42.6	1
d. 1234	3

The mantissa of a logarithm

The mantissa is the decimal part of a logarithm. it can be obtained using the four-figure tables. An excerpt of the four-figure logarithm table is given below.

Examples:

Using the four-figure logarithm table,

1. Find the logarithm of 1000
2. Find the logarithm of 9000
3. Find the logarithm of 46 200
4. Find the logarithm of 46.2
5. Find the logarithm of 1234

Solution:

1. By standard form $1000 = 1.000 \times 10^3$

Hence the characteristic $= 3$.

Or by observation, since 1000 has four digits, then the characteristic $= 3$.

To find the mantissa, we look for "10" under "0" difference "0" using our four-figure log table and so we obtain 0000.

Therefore $\log 1000 = 3.0000$

2. By standard form $9000 = 9.000 \times 10^3$

Hence the characteristic $= 3$.

Or by observation, since 9000has four digits, then the characteristic $= 3$.

To find the mantissa, we look for "90" under "0" difference "0" using our four-figure log table and so we obtain 9542.

Therefore $\log 9000 = 3.9542$

3. By standard form $46\ 200 = 4.6200 \times 10^4$

Hence the characteristic $= 4$.

Or by observation, since $46\ 200$has five digits, then the characteristic $= 4$.

To find the mantissa, we look for "46" under "2" difference "0" using our four-figure log table and so we obtain 6646.

Therefore $\log 46\ 200 = 4.6646$

4. By standard form $46.2 = 4.62 \times 10^1$

Hence the characteristic $= 1$.

To find the mantissa, we look for "46" under "2" difference "0" using our four-figure log table and so we obtain 6646.

Therefore $\log 46.2 = 1.6646$

5. By standard form $1234 = 1.234 \times 10^3$

Hence the characteristic $= 3$.

To find the mantissa, we look for "12" under "3" difference "4" using our four-figure log table and so we obtain 0903.

Therefore $\log 1234 = 3.0903$

TO FIND A NUMBER GIVEN ITS LOGARITHM

To find a number given its logarithm, we simply look at the number which corresponds to the mantissa from a table of anti-logarithms.

For example, if the logarithm of 46 200 $= 4.6646$

To get the number “46 200” back, we simply check for the anti-log of the mantissa, that is, 0.6646 (because $4.6646 = 4 + 0.6646$) by first converting the mantissa into a standard form to have 6.646×10^{-1} and then check for the antilog of 66 under 4 difference 6 which gives us "4619”.

Hence the antilog of $4.6646 = .4619 \times 10^{4}$

LAWS OF LOGARITHMS

The three fundamental laws of Indices can be stated in their equivalent logarithmic form:

1. $x^{a} \times x^{b} = x^{a+b}$

$\Longrightarrow \mathbf{log}(\boldsymbol{MN}) = \mathbf{log}\,\boldsymbol{M} + \mathbf{log}\,\boldsymbol{N}$

2. $x^{a} \div x^{b} = x^{a-b}$

$\Longrightarrow \mathbf{log}(\frac{\boldsymbol{M}}{\boldsymbol{N}}) = \mathbf{log}\,\boldsymbol{M} - \mathbf{log}\,\boldsymbol{N}$

3. $(x^{a})^{b} = x^{ab}$

$\Longrightarrow \mathbf{log}\,\boldsymbol{M^{p}} = \boldsymbol{p} \times \mathbf{log}\,\boldsymbol{M}$

PROOF:

1. From the first law of Indices: $x^{a} \times x^{b} = x^{a+b}$

Let $M = x^{a}\ and\ N = x^{b}$

$\Longrightarrow \log_{x} M = a\ \ and\ \log_{x} N = b$ {by the definition of logarithm}

Thus, $MN = x^{a} \times x^{b}$

$\Longrightarrow MN = x^{a+b}$

Applying the definition of logarithm, we have:

$\log_x(MN) = a + b$

Recalling that, $\log_x M = a \ \ and \ \ \log_x N = b$

$\Longrightarrow \log_x(MN) = \log_x M + \log_x N$

Since x can take any value, hence the law:

$\mathbf{log}(\boldsymbol{MN}) = \mathbf{log}\,\boldsymbol{M} + \mathbf{log}\,\boldsymbol{N}$is true for any base.

2. From the second law of Indices: $x^a \div x^b = x^{a-b}$

Let $M = x^a \ and \ N = x^b$

$\Longrightarrow \log_x M = a \ \ and \ \ \log_x N = b$ {by the definition of logarithm}

Thus, $\frac{M}{N} = x^a \div x^b$

$$\Longrightarrow \frac{M}{N} = x^{a-b}$$

Applying the definition of logarithm, we have:

$$\log_x\left(\frac{M}{N}\right) = a - b$$

Recalling that, $\log_x M = a \ \ and \ \ \log_x N = b$

$$\Longrightarrow \log_x\left(\frac{M}{N}\right) = \log_x M - \log_x N$$

Since x can take any value, hence the law:

$\mathbf{log}(\frac{\boldsymbol{M}}{\boldsymbol{N}}) = \mathbf{log}\,\boldsymbol{M} - \mathbf{log}\,\boldsymbol{N}$is true for any base.

3. From the third law of Indices: $(x^a)^b = x^{ab}$

Let $M = x^a$and $p = b$

$\Longrightarrow \log_x M = a$ {by the definition of logarithm}

Thus, $M^p = (x^a)^b$

$\Longrightarrow \; M^p = x^{ab}$

Applying the definition of logarithm, we have:

$\log_x \; M^p = ab$

Recalling that, $\log_x M = a \; \; and \; p = b$

$\Longrightarrow \log_x \; M^p = (\log_x M) \times p$

Since x can take any value, hence the law:

$\mathbf{\log \boldsymbol{M^p} = \boldsymbol{p} \times \log \boldsymbol{M}}$is true for any base.

Hence from the above laws, it can be deduced that:

1. The logarithm of the product of two or more numbers is the sum of their respective logarithms

i.e.$\mathbf{\log(\boldsymbol{MN}) = \log \boldsymbol{M} + \log \boldsymbol{N}}$

2. The logarithm of two numbers which are being divided is the difference between their respective logarithm

i.e.$\mathbf{\log(\frac{\boldsymbol{M}}{\boldsymbol{N}}) = \log \boldsymbol{M} - \log \boldsymbol{N}}$

3. The logarithm of a number which is raised to the power of another number is the product of the number to which it was raised to and the logarithm of the original number

i.e.$\mathbf{\log \boldsymbol{M^p} = \boldsymbol{p} \times \log \boldsymbol{M}}$

SIMILARLY:

Since $\sqrt[p]{M} = M^{1/p}$ {by law of indices}

$$\Longrightarrow \log \sqrt[p]{M} = \log M^{1/p}$$

$$= \frac{1}{p} \times \log M$$

$$= \frac{\log M}{p}$$

KEY CONCEPTS:

1. To multiply two numbers, we add their respective Logs and then compute the antilog.
2. To divide two numbers, we subtract their respective logs and then compute the antilog.
3. To obtain the result if a number is raised to a power, we multiply the power by the logarithm of the number and then compute the antilog.
4. To obtain the root of any number, we divide the logarithm of the number by its root and then compute the antilog.

MULTIPLICATION AND DIVISION OF LOGARITHMS OF NUMBERS GREATER THAN 1

To multiply and divide numbers, we first express them as logarithms and then apply the above concepts (1) and (2). We use the logarithm tables to change the numbers to logarithms. Finally, we use the anti-logarithm table to change the logarithms back to numbers.

Example:

Using logarithm tables, evaluate the following

a) 76.7×308.2

b) 12.34×139.7

c) $87.42 \div 28.79$

d) $\frac{76.7 \times 308.2}{8.04}$

Solution:

a)

No.	Log.
76.7	1.8848_{+}
308.2	2.4889
	4.3737

Antilog of $4.3737 = 23640$

Hence, $76.7 \times 308.2 = 23640$

To check: Using Calculators;

$76.7 \times 308.2 = 23638.94$which is approximately 23640

b)

No.	Log.
12.34	1.0912_{+}
139.7	2.1453
	3.2365

Antilog of $3.2365 = 1724$

Hence, $12.34 \times 139.7 = 1724$

To check: Using Calculators;

$12.34 \times 139.7 = 1723.898$which is approximately 1724

No.	Log.

c)

87.42	1.9416_{-}
28.79	1.4593
	0.4823

Antilog of $0.4823 = 3.036$

Hence, $87.42 \div 28.79 = 3.036$

To check: Using Calculators;

$87.42 \div 28.79 = 3.036471$ which is approximately 3.036

d)

No.	Log.
76.7	1.8848_{+}
308.2	2.4889
	4.3737_{-}
8.04	0.9053
	3.4684

Antilog of $3.4684 = 2940$

Hence, $\frac{76.7 \times 308.2}{8.04} = 2940$

To check: Using Calculators;

$\frac{76.7 \times 308.2}{8.04} = 2940.1667$ which is approximately 2940

<u>POWERS AND ROOTS OF LOGARITHMS OF NUMBERS GREATER THAN 1</u>

To calculate powers and roots of numbers, we first express them as logarithms and then apply the concepts (3) and (4). We use the logarithm tables to change the numbers to logarithms. Finally, we use the anti-logarithm tables to change the logarithms back to numbers.

<u>Example</u>:

Using logarithm tables, evaluate the following:

a) $(3.55)^4$

b) $\sqrt[5]{40\ 000}$

c) $\frac{\sqrt{94100}\times 38.2}{(5.68)^3\times 8.14}$

d) $\sqrt[3]{20.45}$

Solution:

a)

Solution:

a)

No.	Log.
76.7	1.8848_+
308.2	2.4889
	4.3737

Antilog of $4.3737 = 23640$

Hence, $76.7 \times 308.2 = 23640$

To check: Using Calculators;

b)

Antilog of $4.3737 = 23640$

Hence, $76.7 \times 308.2 = 23640$

To check: Using Calculators;

$76.7 \times 308.2 = 23638.94$which is approximately 23640

c)

No.	Log.
12.34	1.0912_{+}
139.7	2.1453
	3.2365

Antilog of $3.2365 = 1724$

Hence,

$12.34 \times 139.7 = 1724$

To check:

Using Calculators;

$12.34 \times 139.7 = 1723.898$which is approximately 1724

d)

No.	Log.
87.42	1.9416_{-}
28.79	1.4593
	0.4823

Antilog of $0.4823 = 3.036$

Hence,

$87.42 \div 28.79 = 3.036$

To check:

Using Calculators;

$87.42 \div 28.79 = 3.036471$ which is approximately 3.036

No.	Log.
76.7	1.8848_{+}
308.2	2.4889
	4.3737_{-}

8.04	0.9053
	3.4684

e) Antilog of $3.4684 = 2940$

Hence, $\frac{76.7 \times 308.2}{8.04} = 2940$

To check: Using Calculators;

$\frac{76.7 \times 308.2}{8.04} = 2940.1667$ which is approximately 2940

ARITHMETIC OF FINANCE

SIMPLE INTEREST

When money is borrowed, the lender expects to receive a charge from the borrower as compensation. The compensation is generally called INTEREST.

The original amount lent out is referred to as PRINCIPAL.

The period (either in years, months, weeks, etc.) for which the money borrowed is to be used by the borrower is called TIME, while the amount of interest in terms of certain percentage of the original amount lent out is called RATE.

The interest is often expressed as a percentage of the amount of money borrowed. If the interest is calculated on the principal and added at the end of an agreed time interval, it is termed SIMPLE INTEREST.

The simple interest is found using the formula;

$$I = \frac{P \times R \times T}{100}$$

Where, $P = principal,$

$$R = rate, \;\; T = time \quad and$$

$$I = interest.$$

Note: the sum of the PRINCIPAL and the INTEREST is called THE AMOUNT.

$$That\ is: \qquad A = P + I$$

Example 1:

Find the simple interest on a loan of ₦3000 for 2 years at a rate of 5% per annum.

Example 2:

Calculate the time at which ₦1200 will yield an interest of ₦192 at 8% per annum.

Example 3:

Calculate the amount if the simple interest is paid yearly at 12% per annum for 3 years on a principal of ₦$250\ 000$.

COMPOUND INTEREST

Compound interest is the interest added to the principal at regular intervals so that the principal grows as interest is added. An example of compound interest is savings account interest.

To find compound interest, the amount at the end of each year must be calculated on a systematic basis and then used as the principal for the next year.

Example 4:

Calculate the compound interest on ₦$250\ 000$ in 3 years at 4% per annum.

Example 5:

A man saved ₦1000. Calculate the amount he will get in 1 year if the interest rate is 4.2% compounded quarterly.

THE COMPOUND INTEREST FORMULA

From; $A = P + I$

$$Since;\ \ I = \frac{PRT}{100}$$

$$\Longrightarrow\ \ A = P + \frac{PRT}{100}$$

Factorizing, we have:

$$\Longrightarrow \quad A = P\left(1 + \frac{RT}{100}\right)$$

Since the amount is calculated at the end of every interval. Hence, $T = 1$

$$So, \qquad A = P\left(1 + \frac{R}{100}\right)$$

Therefore, for n number of years or interval, the total amount will be;

$$A = P\left(1 + \frac{R}{100}\right)^n$$

Hence, the compound interest;

$$C.I = AMOUNT - PRINCIPAL$$

$$i.e. \qquad P\left(1 + \frac{R}{100}\right)^n - P$$

Example 6:

Calculate the compound interest on ₦$250\ 000$ in 3 years at 4% per annum.

Example 7:

A man saved ₦1000. Calculate the amount he will get in 1 year if the interest rate is 4.2% compounded quarterly.

DEPRECIATION

Many items such as cars and phones lose value as time passes. This lose in value is called **depreciation**.

Depreciation is a situation in which an item loses value over a period of time. It is usually given as a percentage of the value of the item at the beginning of the year.

Example 8:

A small electricity generating set costing ₦20,000 loses its value by 25% during the first year and 15% in the second year. Calculate its current value at the end of the second year.

Example 9:

A radio costing ₦16,000 depreciates by 20% in the first year and by 18% in the second year. Find its value after 2 years.

INFLATION

Due to rising prices, money loses some of its value as time passes. Loss in value of money is called **Inflation.** Inflation is usually given as the percentage increase in the cost of buying things from one year to the next.

For example, suppose the rate of inflation is 20%. Then a phone that cost ₦50,000 a year ago will now cost:

$$\text{₦}50{,}000 + \left(\frac{20}{100} \times \text{₦}50{,}000\right)$$

$$= \text{₦}60{,}000$$

Thus, money has lost some of its value since it now costs more to buy the same thing.

Example 10:

In a certain year, a new car cost ₦$16,000,000$. If the rate of inflation was 20% for the next two years, what would the same car cost at the end of this period?

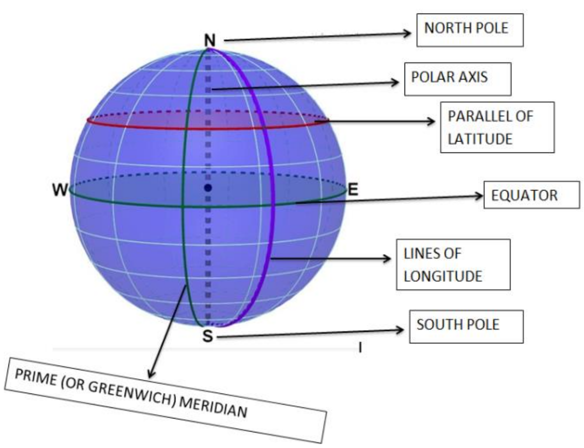
LATITUDE AND LONGITUDE

THE EARTH AS A SPHERE

The earth is not exactly a sphere because the radius of the earth is not constant. Its radius varies between 6360 km and 6380 km. But in the field of mathematics and other related disciplines, the earth is usually assumed to be a sphere of radius varying between 6300 km and 6400 km approximately.

The earth is also called the GLOBE. It is not purely round, neither is it oval in shape like an egg. It looks more similar like an orange.

Below is a view of a model of the earth;



MAJOR AXIS OF THE EARTH

The earth has two major axes: The North-South axis and the East-West axis.

The North-South (NS) axis, called the **polar axis,** is the imaginary line through the centre of the earth from the North Pole to the South Pole.

The East-West (ES) axis is the imaginary line that runs in the East-West direction dividing the whole earth into two hemispheres; the northern hemisphere and the southern hemisphere.

Note: A hemisphere is simply the half-part of a sphere.

The two axes, North-South and East-West axes, intersect each other at right angles at an imaginary point, O, called the centre of the earth.

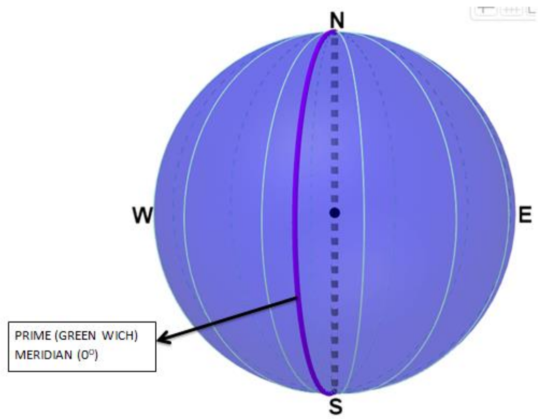
The earth rotates about its polar axis (NS pole) every 24 hours to cause day and night.

The northern part of the polar axis is called the North Pole, while the southern end point of the axis is called the South Pole.

LINES OF LATITUDE AND LONGITUDE

The imaginary lines drawn on the surface of the earth are called lines of latitude and longitude. They are used to specify the position of points on the surface of the earth.

LINES OF LONGITUDE



THE LONGITUDES

If the earth is cut through its polar axis (i.e. N-S) as shown above, the circles so formed on the earth's surface are called **lines of longitude.** These lines of longitude are also called **the great circles**, otherwise known as **the meridians.** They vary from $180°E$ to $180°W$.

So, we can say that the imaginary lines drawn that run vertically from the North Pole to the South Pole are called lines of longitude.

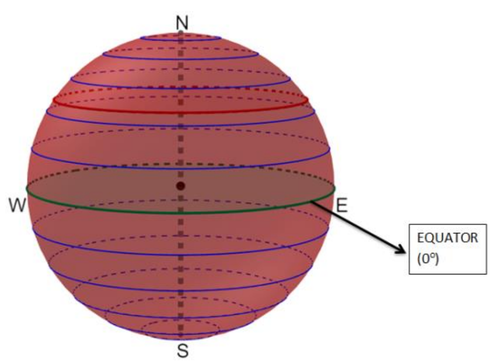
The lines of longitude or meridians are great circles that converge at the poles; thus meridians are not parallel lines.

There is a particular meridian called **the prime meridian** which passes through Greenwich in London, and also through the town of Tema in Accra Ghana. For this reason, it is also called the **Greenwich meridian.**

The Greenwich meridian has a longitude of $0°$ on the earth's surface, so it is used as the standard from which the positions of other meridians are measured in degrees east or west.

All longitudes to the left of the prime meridian are measured from the prime meridian $(0°)$ to $180°$ west and those to the right of the prime meridian are measured from the prime meridian $(0°)$ to $180°$ east.

LINES OF LATITUDE



THE PARALLEL OF LATITUDES

If the earth is cut by horizontal planes at right angle to the polar axis, the circles so formed on its surface are known as **lines of latitude.** The largest of these circles is called the EQUATOR.

The equator is a great circle which divides the earth into two equal halves; the northern and southern hemispheres. The equator is the only line of latitude whose centre is also the centre of the earth.

All of the other lines of latitude are small circles, often called parallel of latitude since they are parallel to each other.

Note: The farther the circles of latitude are from the equator, the smaller they become.

The latitude of a place is measured in degrees North or South of the equator which varies from $90°N$ to $90°S$.

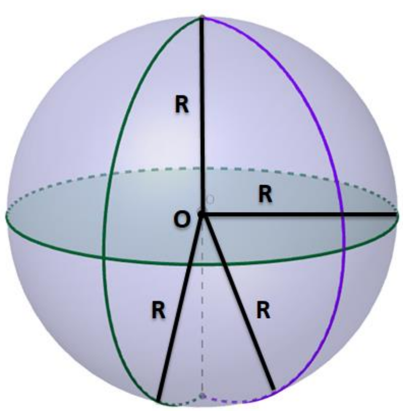
Just as we have the Greenwich meridian as the reference longitude, the equator is the reference latitude and it is designated as latitude $0°$.

GREAT AND SMALL CIRCLES

GREAT CIRCLES:

A circle with the largest radius which is drawn around a sphere with a common centre and equal radius as that of the sphere is called a great circle.

From the the diagram below,

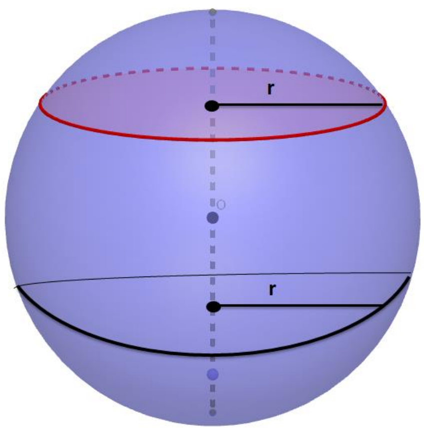


It can be seen that all meridians are great circles including the equator.

SMALL CIRCLES

All circles on a sphere of radius say, r, whose centres and radii do not pass through the centre of the sphere are called small circles.

From the diagram below, it can be seen that all the parallel of latitudes apart from the equator are small circles.



LOCATION OF POINTS ON THE EARTH'S SURFACE

The position of a point on the earth's surface can be completely defined by its latitude and longitude as an ordered pair in the form (Latitude, Longitude).

The point or place on the earth's surface is the intersection of latitude and a longitude passing through the point or place.

Example:

Locate the position of the following points on the earth's surface

a) $A\ (60°N, 60°E)$

b) $B\ (60°N, 75°E)$

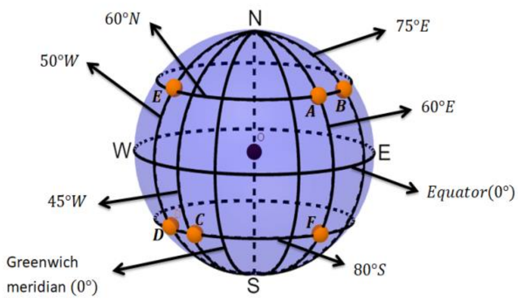
c) $C\ (80°S, 45°W)$

d) $D\ (80°S, 50°W)$

e) $E\ (60°N, 50°W)$

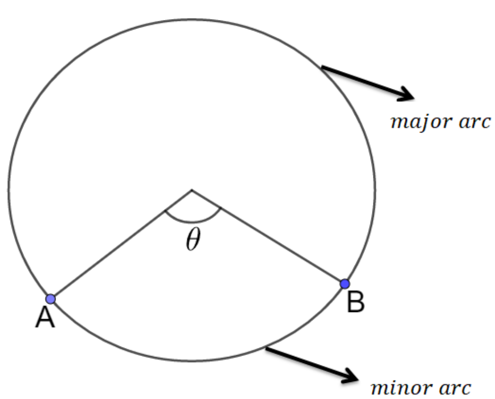
f) $F\ (80°S, 60°E)$

Solution:



ANGULAR DIFFERENCE BETWEEN TWO PLACES ON THE EARTH'S SURFACE

The angle subtended at the centre of a great or small circle by the minor arc joining two places on the great or small circle respectively is called the angular difference between the two places.



$\boldsymbol{\theta}$ ***is the angular difference***

The angular difference can be considered in two parts;

1. When the two places are on the same parallel of latitude
2. When the two places are on the same meridian (or longitude).

WHEN THE TWO PLACES ARE ON THE SAME PARALLEL OF LATITUDE

Example:

Find the angular difference between the following pairs of places on the surface of the earth.

a) $A\ (62°N, 60°E)$ and $G\ (62°N, 52°W)$

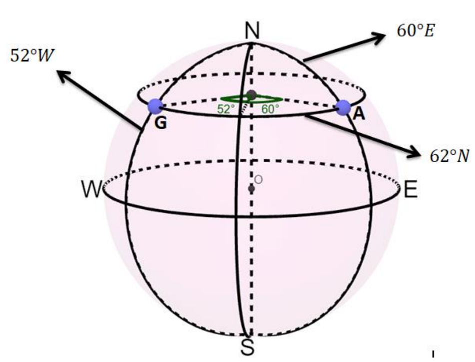
b) $J\ (60°S, 47°E)$ and $K\ (60°S, 79°E)$

c) $M\ (50°N, 30°W)$ and $N\ (50°N, 100°W)$

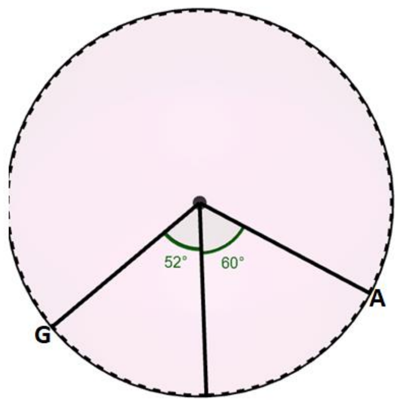
d) $X\ (77°S, 22°W)$ and $Y\ (77°S, 45°E)$

Solution:

a) $A\ (62°N, 60°E)$ and $G\ (62°N, 52°W)$

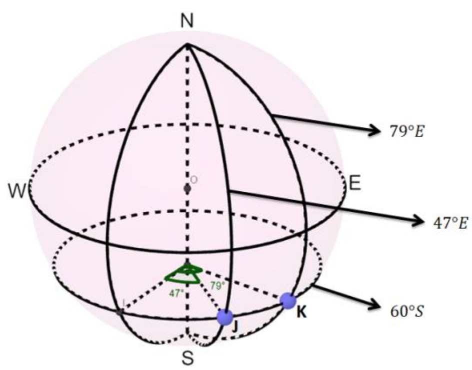


Bringing out the circle, we have:

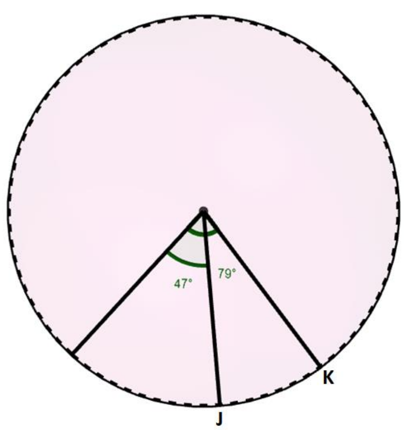


Angular difference $= 52° + 60° \qquad = 112°$

b) $J\ (60°S, 47°E)$ and $K\ (60°S, 79°E)$

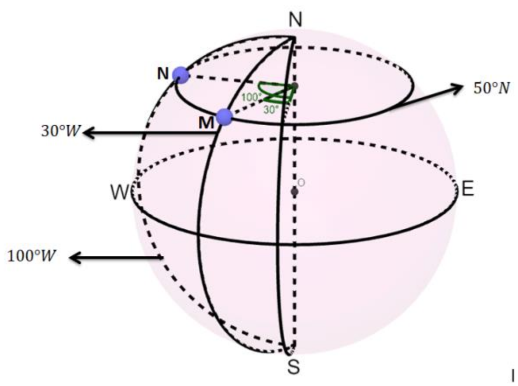


Bringing out the circle, we have:

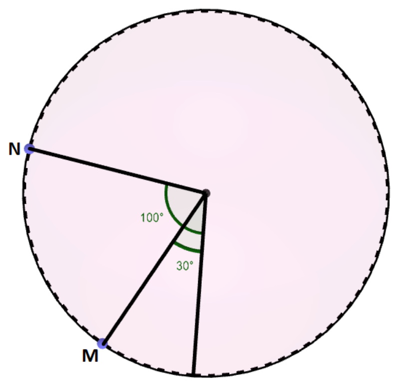


Angular difference $= 79° - 47° \qquad = 32°$

c) $M\ (50°N, 30°W)$ and $N\ (50°N, 100°W)$

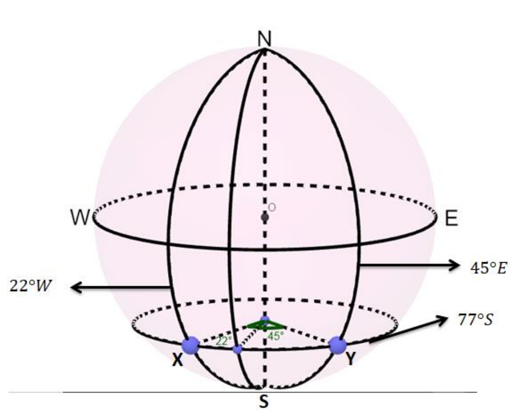


Bringing out the circle, we have:

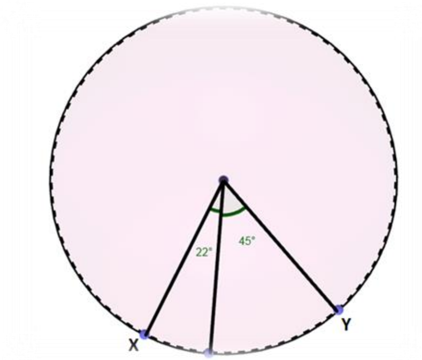


Angular difference $= 100° - 30° \quad = 70°$

d) $X\ (77°S, 22°W)$ and $Y\ (77°S, 45°E)$



Bringing out the circle, we have:



Angular difference $= 22° + 45° \qquad = 67°$

WHEN THE TWO PLACES ARE ON THE SAME MERIDIAN (OR LONGITUDE)

Example:

Find the angular difference between the following pairs of places on the surface of the earth.

a) $A\ (62°N, 60°E)$ and $G\ (49°S, 60°E)$

b) $J\ (40°S, 60°E)$ and $K\ (60°S, 60°E)$

c) $A\ (70°N, 40°W)$ and $B\ (87°S, 40°W)$

d) $C\ (65°S, 40°W)$ and $D\ (87°S, 40°W)$

RADIUS OF THE EARTH'S SURFACE AND RADIUS OF PARALLEL OF LATITUDES

The earth is approximately a sphere. Its radius has been calculated and is approximately equal to 6400km. In performing any calculation, we use the radius of the earth's surface as $R = 6400km$, except stated otherwise.

Note: The radius of the earth's surface is distance from the centre of the sphere to any part of the sphere. It is the radius of all great circles (longitudes and equator). It is denoted with R.

Other small circles on the earth's surface also have their separate radii. We denote the radius of any small circle with r (small letter, r) to distinguish it from the radius of the great circles. The radius of the small circles is also called the radius of parallel of latitudes.

Relationship between the radius of the parallel of latitude, (r) and the radius of the earth, (R):

Let's consider a point say, P with latitude $\propto°$ North or South on the earth's surface as shown below:

If the radius of the small circle of latitude $\propto°$ North or South is r and R is the radius of the earth.

Considering the $\triangle\, OPT$, using SOHCAHTOA;

$$\cos\alpha = \frac{r}{R}$$

$$\Longrightarrow r = R\cos\alpha$$

Hence, the radius of the parallel of latitude is;

$$r = R\cos\alpha$$

Where, $r =$ radius of the parallel of latitude

$R =$ Radius of the earth and

$\alpha =$ The latitude itself

DISTANCES ON GREAT CIRCLES

If there are two places on different locations on the earth surface, their distance apart can be calculated. Such distance apart can either be on a great circle or on a small circle.

For a distance to be on a great circle, the two positions connecting the distance must either be on the equator or on the same longitude (either east or west) but on different longitudes or latitudes as the case may be.

In calculating the distance along great circles, what is required is to calculate the length of the arc on the sphere of radius, R, which is the radius of the earth using the formula:

$$Distance = \frac{\theta}{360°} \times 2\pi R$$

Where, θ is the angular difference.

Example:

Find the distance between the points $P\ (Lat. 46°N, Long. 70°E)$ and $Q\ (Lat. 60°S, Long. 70°E)$ on the surface of the earth. Take $\pi = 3.142$

Solution:

DISTANCES ON SMALL CIRCLES

For a distance to be on a small circle, the two positions connecting the distance must be on the same latitude either North or South (apart from the equator, which itself is also a great circle).

In calculating the distance along small circles, what is required is to calculate the length of the arc on the small circle with radius, r, using the formula:

$$Distance = \frac{\theta}{360^\circ} \times 2\pi r$$

Where, $r = R\cos\alpha$

$\alpha =$ The latitude itself,

θ is the angular difference, and

$R =$ Radius of the earth