WEEK ONE:

TOPIC: GEOMETRY

SUB-TOPIC: MENSURATION: (i)Length of arc of a circle, (ii)Perimeter of sectors and segments, (iii)Area of sectors and segments of a circle, (iv)Relationship between the sector of a circle and the surface area of a cone.

What is Mensuration?

Mensuration is the branch of mathematics that studies the measurement of geometric figures and their parameters like length, volume, shape, surface area, lateral surface area, etc.

LENGTH OF AN ARC OF A CIRCLE

An arc length is the distance between two points along a section of a curve.
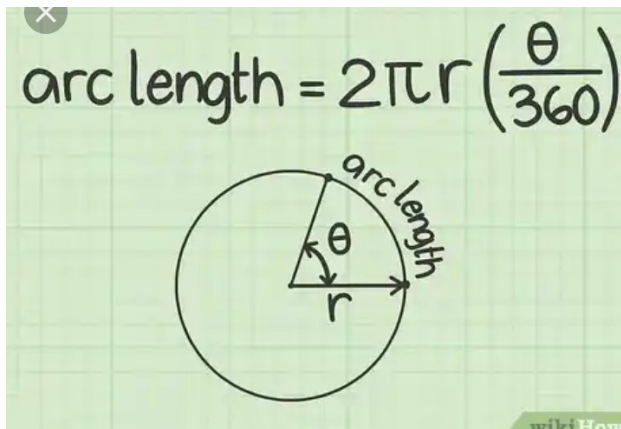
Arc Length Formula (if θ is in degrees): L = 2 π r (θ/360°)

L = Length of an Arc

θ = Central angle of Arc

r = Radius of the circle

Question 1: Calculate the length of an arc if the radius of an arc is 8 cm and the central angle is 40°.

Solution:

Radius, r = 8 cm, Central angle, θ = 40°

Arc length = 2 π r × (θ/360°), So, L = 2 × π × 8 × (40°/360°) = 5.582 cm
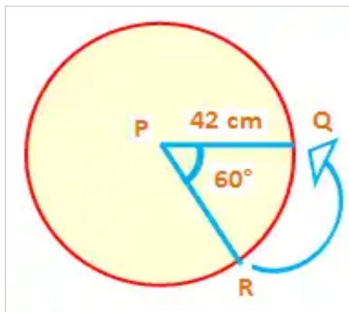
 PERIMETER OF SECTOR AND SEGMENT

The perimeter of a sector is the distance around a sector. We can calculate the perimeter of a sector by adding together the lengths of the two radii and the arc length of the sector.

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It is given as $2r + 2\pi r \times (\theta/360°$

Example 1 :

Find the perimeter of the sector PQR shown below.

Length of the arc is

$L = \theta/360° \times 2\pi r$

Substitute $\theta = 60°$, r = 42 and $\pi = 22/7$.

$L = 60°/360° \times 2 \times 22/7 \times 42$

$L = 1/6 \times 2 \times 22 \times 6$

$L = 1 \times 2 \times 22$

$L = 44$ cm

Perimeter of sector is

$= L + 2r$

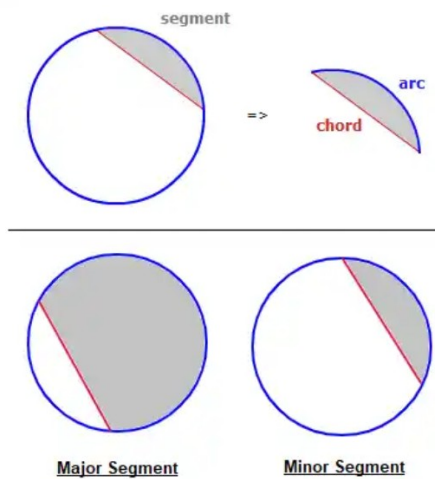Substitute L = 44 and r = 42.

$= 44 + 2(42)$

$= 44 + 84 = 128$ cm, So, length of the arc is 128 cm.

PERIMETER OF A SEGMENT:

Perimeter of segment = LENGTH OF CHORD + LENGTH OF ARC
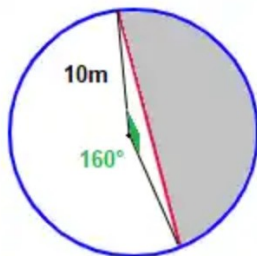
LENGTH OF ARC + LENGTH OF CHORD $= (\frac{\theta}{360} \times 2\pi r) + (2r \sin(\frac{\theta}{2}))$
with degree measure.

AND

LENGTH OF ARC + LENGTH OF CHORD $= r\theta + (2r \sin(\frac{\theta}{2}))$
with radian measure.

These formulas can be used to obtain a measure of segment perimeter length.

Example:

Perimeter of Segment $= \left(\frac{160}{360} \times 2\pi(10) \right) + \left(2(10) \times \sin\left(\frac{160}{2}\right) \right)$

$= \left(\frac{4}{9} \times 20\pi \right) + \left(20 \times \sin(80) \right)$

$= \quad 27.93 + 19.70 \quad = \quad 47.63 \text{m}$

Relationship between the sector of a circle and the surface area of a cone.
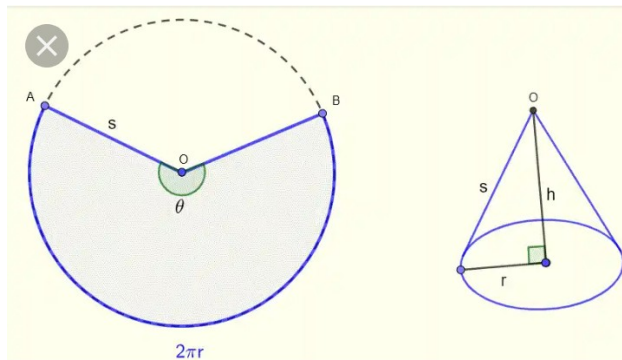
WEEK TWO:

TOPIC: GEOMETRY

SUB-TOPIC: MENSURATION: (i)Surface area and volume of solid shapes:-cube,cuboids,cylinder,cone, prism, pyramid, sphere, hemisphere, frustum, compound shapes

CUBE:

Total Surface area of a cube is 6×L×L

The volume of a cube can be found by multiplying the edge length three times

Volume of a Cube = L×L×L

CUBOID:

The total surface area of a cuboid is the sum of all its surfaces. In order to find the total surface area of a cuboid, we need to add the area of all the 6 rectangular faces. The formula for the total surface area of a cuboid is 2 (lw + wh + lh) where l = length, w = width, and h = height of the cuboid

The formula of volume of a cuboid is = Length × Width × Height.

CYLINDER

The formula to calculate the total surface area of a cylinder is expressed as, total surface area of cylinder = 2πr(r + h).

The volume of a cylinder, V=2πrh

CONE

The Total Surface Area of a cone, T = πr(r + l) square units.

The formula for the volume of a cone is V=1/3hπr².

PRISM

The formula for the surface area of a prism is obtained by taking the sum of (twice the base area) and (the lateral surface area of the prism). The surface area of a prism is given as S = (2 × Base Area) + (Base perimeter × height) where "S" is the surface area of the prism.

Volume:

Any prism volume is V = BH where B is area of base and H is height of prism, so find area of the base by B = 1/2 h(b1+b2), then multiply by the height of the prism.

$$V = \frac{b \cdot h \cdot l}{2}$$

$$V = \pi r^2 h$$

$$V = \frac{\pi r^2 h}{3}$$

$$V = \frac{l \cdot w \cdot h}{3}$$

$$V = \frac{4}{3}\pi r^3$$

$$V = l \cdot w \cdot h$$
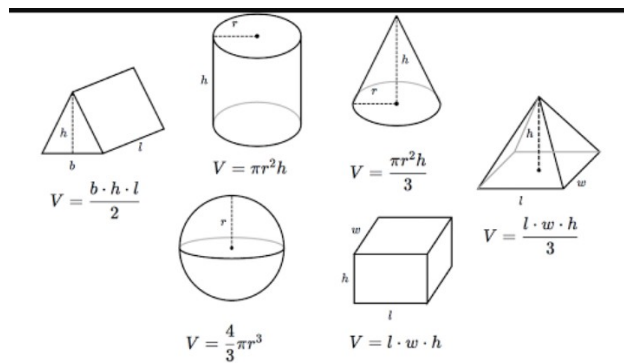
PYRAMID:

A pyramid is a polyhedron formed by connecting a polygonal base and an apex. The basic formula for pyramid volume is the same as for a cone: volume = (1/3) * base_area * height , where height is the
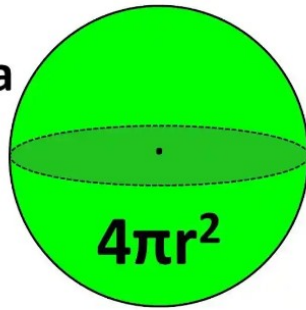
Pyramid		Volume
Triangular Pyramid		$V = \frac{1}{3} \times B \times h$ $= \frac{1}{3} \times \frac{1}{2} \times bH \times h$ $V = \frac{1}{6} BHh$
Square Pyramid		$V = \frac{1}{3} \times B \times h$ $V = \frac{1}{3} a^2 h$
Rectangular Pyramid		$V = \frac{1}{3} \times B \times h$ $V = \frac{1}{3} \times lw \times h$ $V = \frac{1}{3} lwh$
Pentagonal Pyramid		$V = \frac{1}{3} \times B \times h$ $= \frac{1}{3} \times \frac{5}{2} Sa \times h$ $V = \frac{5}{6} Sah$
Hexagonal Pyramid		$V = \frac{1}{3} \times B \times h$ $V = \frac{1}{3} \times 3aS \times h$ $V = aSh$

height from the base to the apex.

SPHERE:

Surface Area of a Sphere

$$4\pi r^2$$

Name of the Solid	Figure	Lateral/Curved Surface Area	Total Surface Area	Nomenclature
Cube		$4a^2$	$6a^2$	a : side of cube
Cuboid		$2h\,(l+b)$	$2(lb + bh + hl)$	l : length b : breadth h : height
Cone		$\pi r l$	$\pi r(l+r)$	r : radius of base h : height l : slant height
Cylinder		$2\pi rh$	$2\pi r(r+h)$	r : radius of base h : height
Sphere		$4\pi r^2$	$4\pi r^2$	r : radius
Hemisphere		$2\pi r^2$	$3\pi r^2$	r : radius
Prism		Perimeter of base x height	Lateral Surface area +2(area of the base)	—
Pyramid		$\frac{1}{2}$(Perimeter of base) x slant height	Lateral Surface area + area of the base	—

SOLVED EXAMPLES:

and this needs thorough practice and precision.

Solved Examples for You

Question 1: Shreya made a cylindrical bird-bath with a hemispherical depression at the upper end. The radius of the circular shaped top is 30cm and height of the cylinder is 1.45 m. Find the total surface area of the bird bath?

Answer : The radius of both cylinder and hemisphere are common, hence taken as r = 30cm = 0.3m. Height (h) of the cylinder = 1.45 m.

TSA of the Bird-Bath = CSA of Cylinder + CSA of the Hemisphere

= 2 πrh + 2πr2

= 2 π r(h + r)

= 2 × (22/7) × 0.30 (1.45 + 0.30) m

= 3.3 m2

Q.2: What is the surface area of a cuboid with length, width and height equal to 4.4 cm, 2.3 cm and 5 cm, respectively?

Solution: Given, the dimensions of cuboid are:

length, l = 4.4 cm

width, w = 2.3 cm

height, h = 5 cm

Surface area of cuboid = 2(wl+hl+hw)

= 2·(2.3·4.4+5·4.4+5·2.3)

= 87.24 square cm.

Q.3: What is the volume of cylinder whose base radii are 2.1 cm and height is 30 cm?

Solution:

Radius of bases, r = 2.1 cm

Height of cylinder = 30 cm

Volume of cylinder = πr2h = π·(2.1)2·30 ≈ 416

WEEK THREE:

TOPIC: STATISTICS

SUB-TOPIC: Data Presentation: Revision on collection, tabulation and presentation of data, Frequency distribution.

What is Statistics? This is a branch of mathematics dealing with the collection, analysis, interpretation, and presentation of masses of numerical data.

Data:There are two possible ways in which data can be classify and these are Grouped and Ungrouped data.Data is/are sometimes referred to as information. Although they differs in so many ways,i.e,information is wider than data,hence data is found under the information of a certain event.

It is a common practice to present data in frequency tables. Frequency tables are used for summarising data before analysis.

Example 1: The weight of some students in SSS1 class in Good Shepherd Schools are as listed below:
55,57,57,59,50,55,61,61,55,57,
57,57,59,55,55,50,55,55,50,57

57,57,59,50,50,55,57,57,55,5050,5
0,55,57,61,57,59,61,59,55

,
61,55,57,55,50,61,59,55,57,61

Prepare the frequency distribution table for the information.

Solution:

Weight (x)	Tally	Frequency(f)
50	॥॥॥	8
55	॥॥॥ ॥	11
57	॥॥॥ ॥	11
59	॥॥॥	7
61	॥॥	5

Example 2:Prepare a frequency table,showing the percentage scores of each of the scores obtained in a mathematics test of students in SSS 1 Shephered.The scores are:

9, 7 ,8, 5, 4, 6, 5, 8, 6, 6, 10, 5, 6, 7, 6, 6, 5, 5, 7, 8, 10, 2, 8, 6, 6, 2, 6, 4, 5, 5, 8, 8, 6, 6, 5, 9, 9, 2, 7, 4, 6, 3, 5, 6, 2, 7, 2, 9, 8, 10

Solution

MARKS	TALLY	FREQUENCY	PERCENTAGE%
2	ЖЖ	5	10
3	I	1	02
4	I II	3	06
5	ЖЖIIII	9	18
6	ЖЖI ЖIII	13	26
7	ЖЖ	5	10
8	ЖЖII	7	14
9	I III	4	08
10	I II	3	06

Calculation of Range, Median and Mode of Ungrouped Data

RANGE

The range of a set of numbers is the difference between the largest and the smallest numbers.

Example: Find the range of the following set of scores: 79, 60, 52, 34, 58, 60.

Solution

Arrange the set in rank order: 79, 60, 60, 58, 52, 34

The range is 79 – 34 = 45

THE MEAN

There are many kinds of average. T hemean or arithmetic mean, is the most common kind. If there are n numbers in a set, then

Mean = sum of the numbers in the set/ n

Examples

1)Calculate the mean of the following set of numbers.

176 174 178 181 174

175 179 180 177 182

Solution

Mean = 176 + 174 + 178 + …. + 182/10

= 1776/10 = 177.6

2)Five children have an average age of 7 years 11 months . If the youngest child is not included, the average increares to 8 years 4 months. Find the age of the youngest child.

Solution

Total age of all five children

= 5 x 7 yr 11 mo

= 35 yr 55 mo

= 35 yr + 4 yr 7 mo

= 39 yr 7 mo

Total age of the four older children

= 4 x 8 yr 4 mo

= 32 yr 16 mo

= 32yr + 1 yr 4 mo

= 33 yr 4 mo

Age of youngest child

= 39yr 7 mo – 33 yr 4 mo

= 6 yr 3 mo

Evaluation

1. Find x if the mean of the numbers 13, 2x, 0, 5x and 11 is 9. Also find the range of the set of numbers.

WEEK FOUR

TOPIC: Statistics

SUB-TOPIC: BASIC TOOLS: (i)Linear graph, bar graph and histograms, (ii)Pie chart, (ii)Frequency polygon

BAR CHART:

A bar chart is a graph with rectangular bars. The graph usually compares different categories. A Bar Graph (also called Bar Chart) is a graphical display of data using bars of different heights

A bar chart or bar graph presents data with rectangular bars at heights or lengths proportional to the values they represent.

Examples:
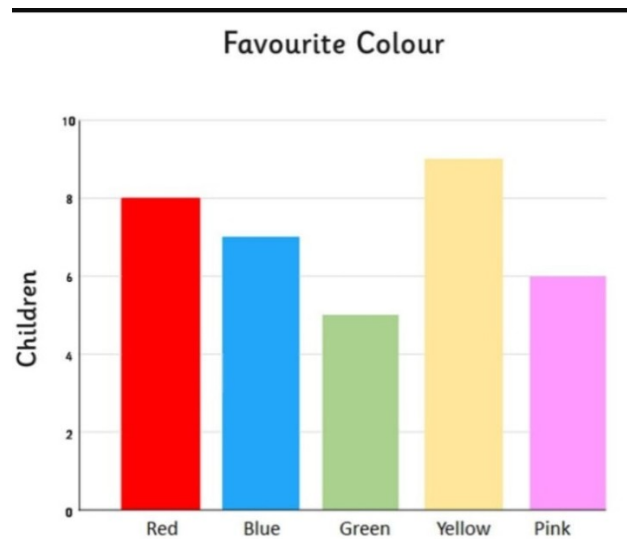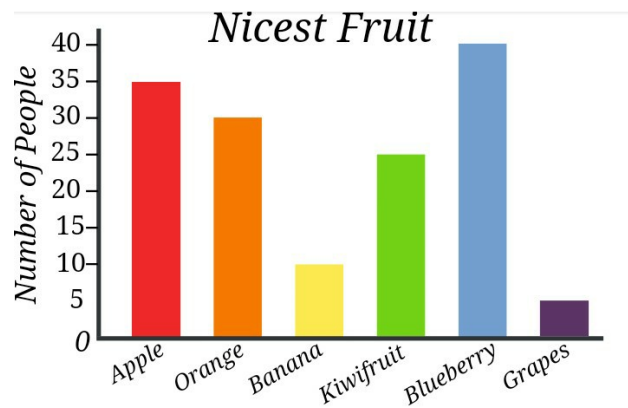
PIE CHART:

A pie chart (or a circle chart) is a circular statistical graphic, which is divided into slices to illustrate numerical proportion. In a pie chart, the arc length of each slice (and consequently its central angle and area) is proportional to the quantity it represents.

Examples:

How Much Water Do We Use?

Shower 16.8%
Toilet 26.7%
Faucet 15.7%
Leaks 13.7%
Clothes Washer 21.7%
Other 5.3%
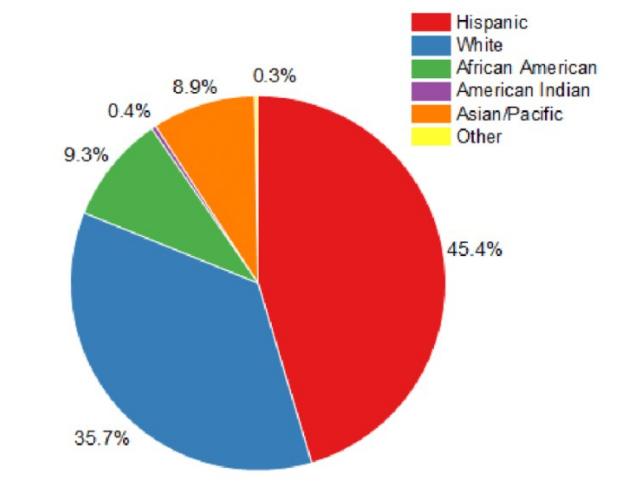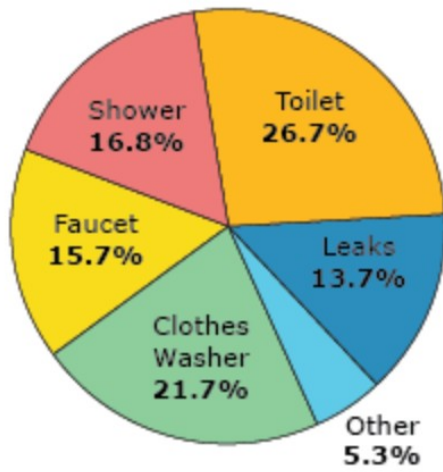
Source: American Water Works Association Research Foundation, "Residential End Uses of Water." 1999

Australia
Other
Canada
UK
USA
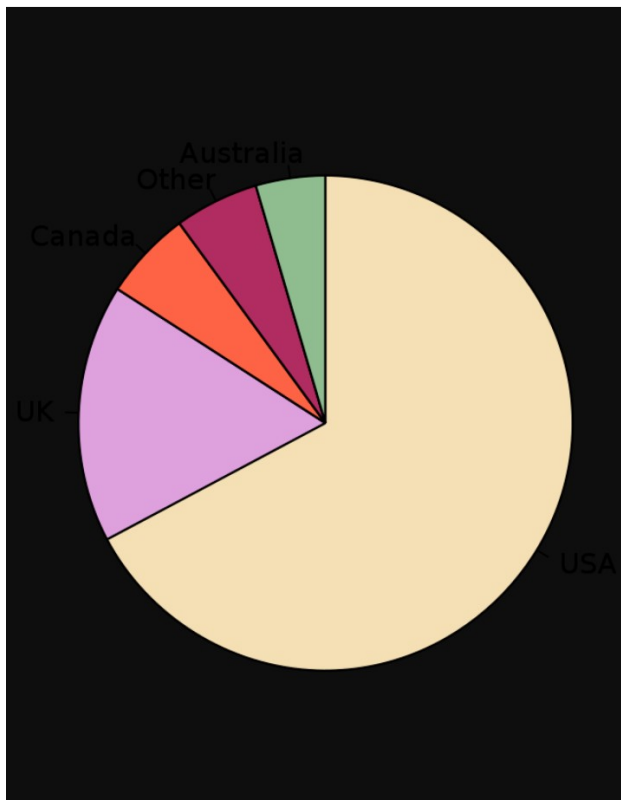
LINE GRAPH:

A line graph is a graphical display of information that changes continuously over time. Within a line graph, there are various data points connected together by a straight line that reveals a continuous change in the values represented by the data points.

Examples:
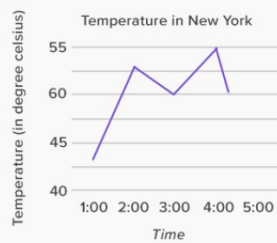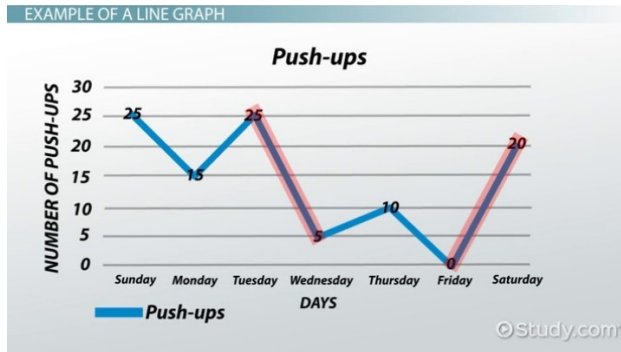
HISTOGRAM:

Histogram: a graphical display of data using bars of different heights. It is similar to a Bar Chart, but a histogram groups numbers into ranges.
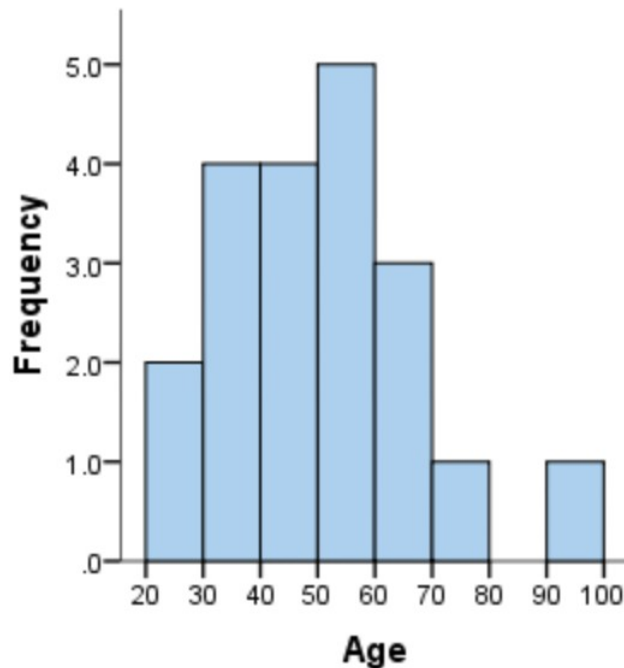
Chart 5.7.1
Distribution of salaries of the employees of ABC Corporation
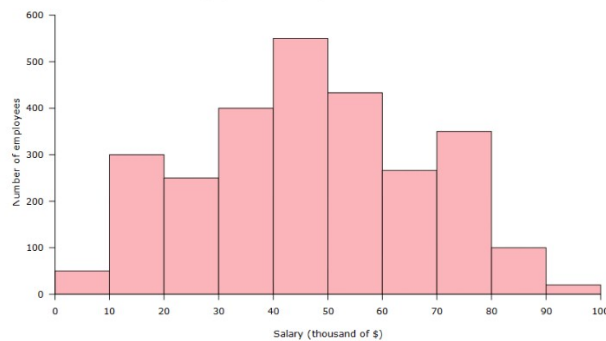
WEEK 6

SEQUENCE AND SERIES

Sequence and series is one of the basic topics in Arithmetic. An itemized collection of elements in which repetitions of any sort are allowed is known as a sequence, whereas series is the sum of all elements. An arithmetic progression is one of the common examples of sequence and series.

A sequence is defined as an arrangement of numbers in a particular order. On the other hand, a series is defined as the sum of the elements of a sequence.   In short, a sequence is a list of items/objects which have been arranged in a sequential way.

A series can be highly generalized as the sum of all the terms in a sequence. However, there has to be a definite relationship between all the terms of the sequence.

The fundamentals could be better understood by solving problems based on the formulas. They are very similar to sets but the primary difference is that in a sequence, individual terms can occur repeatedly in various positions. The length of a sequence is equal to the number of terms and it can be either finite or infinite. This concept is explained in a detailed manner in Class 11 Maths. With the help of definition, formulas and examples we are going to discuss here the concepts of sequence as well as series

Sequence and Series Definition

A sequence is an arrangement of any objects or a set of numbers in a particular order followed by some rule. If a1, a2, a3, a4,   etc. denote the terms of a sequence, then 1,2,3,4, ..denotes the position of the term.

A sequence can be defined based on the number of terms i.e. either finite sequence or infinite sequence.

If a1, a2, a3, a4,   . is a sequence, then the corresponding series is given by

SN = a1+a2+a3 + .. + aN

Note: The series is finite or infinite depending if the sequence is finite or infinite.

Types of Sequence and Series

Some of the most common examples of sequences are:

Arithmetic Sequences

Geometric Sequences

Harmonic Sequences

Fibonacci Numbers

Arithmetic Sequences

A sequence in which every term is created by adding or subtracting a definite number to the preceding number is an arithmetic sequence.

What are Finite and Infinite Sequences and Series?

Sequences: A finite sequence is a sequence that contains the last term such as a1, a2, a3, a4, a5, a6   an. On the other hand, an infinite sequence is never-ending i.e. a1, a2, a3, a4, a5, a6   an ..

Series: In a finite series, a finite number of terms are written like a1 + a2 + a3 + a4 + a5 + a6 +   an. In case of an infinite series, the number of elements is not finite i.e. a1 + a2 + a3 + a4 + a5 + a6 +   an + ..

Geometric Sequences

A sequence in which every term is obtained by multiplying or dividing a definite number with the preceding number is known as a geometric sequence.

Harmonic Sequences

A series of numbers is said to be in harmonic sequence if the reciprocals of all the elements of the sequence form an arithmetic sequence.

Fibonacci Numbers

Fibonacci numbers form an interesting sequence of numbers in which each element is obtained by adding two preceding elements and the sequence starts with 0 and 1. Sequence is defined as, F0 = 0 and F1 = 1 and Fn = Fn-1 + Fn-2

Sequence and Series Formulas

List of some basic formula of Arithmetic progression and Geometric progression are

Arithmetic Progression a, a+d, a+2d,   ,a+(n-1)d

Geometric Progression a, ar, ar2, .,ar(n-1),

, . Common Difference or Ratio Successive term   Preceding term

Common difference = d = a2   a1

Successive term/Preceding term

Common ratio = r = ar(n-1)/ar(n-2)

nth Term of an A.P T_n = a + (n-1)d

Sum of first n term of an A.P $S_n = \frac{n}{2}\{2a + (n-1)d\}\frac{n}{2}\{2a + (n-1)d\}$

nth term of an A,P $T_n = ar^{n-1}r^{n-1}$

Sum of first n term of a G.P $\qquad S_n = \dfrac{a(r^n-1)a(r^n-1)}{r-1 \quad\quad r-1}$ If $r > 1 > 1$,

$$S_n = \dfrac{a(1-r^n)a(1-r^n)}{1-r \quad\quad 1-r} \qquad \text{if } r < < 1$$

*Here, a = first term, d = common difference, r = common ratio, n = position of term, l = last term

Difference between Sequences and Series

Let us find out how a sequence can be differentiated with series.

Sequences	Series
Set of elements that follow a pattern	(1) Sum of elements of the sequence
Order of elements is important	(2) Order of elements is not so important
Finite sequence: 1,2,3,4,5	(3) Finite series: 1+2+3+4+5
Infinite sequence: 1,2,3,4,	(4) Infinite Series: 1+2+3+4+

Sequence and Series Examples

Question 1: If 4,7,10,13,16,19,22   is a sequence, Find: (a) Common difference (b) nth term   (c) 21st term

Solution: Given sequence is, 4,7,10,13,16,19,22

a) The common difference = 7   4 = 3

b) The nth term of the arithmetic sequence is denoted by the term Tn and is given by Tn = a + (n-1)d, where   a   is the first term and d, is the

common difference.

Tn = 4 + (n   1)3 = 4 + 3n   3 = 3n + 1

c) 21st term as: T21 = 4 + (21-1)3 = 4+60 = 64.

Question 2: Consider the sequence 1, 4, 16, 64, 256, 1024 .. Find the common ratio and 9th term.

Solution: The common ratio (r) = 4/1 = 4

The preceding term is multiplied by 4 to obtain the next term.

The nth term of the geometric sequence is denoted by the term Tn and is given by Tn = ar(n-1)

where a is the first term and r is the common ratio.

Here a = 1, r = 4 and n = 9

So, 9th term is can be calculated as T9 = 1* (4)(9-1)= 48 = 65536.

Some common questions on A.P and G.P.

Show that the sequence 7, 11, 15, 19, 23, ... is an Arithmetic Progression. Find its 27th term and the general term.

The 5th term of an Arithmetic Progression is 16 and 13th term of an Arithmetic Progression is 28. Find the first term and common difference of the Arithmetic Progression.

Find the sum of the following Arithmetic series: 1 + 8 + 15 + 22 + 29 + 36 +   to 17 terms

Find the sum of the series: 7 + 15 + 23 + 31 + 39 + 47 +   .. + 255

Find the sum of the first 35 terms of an Arithmetic Progression whose third term is 7 and seventh term is two more than thrice of its third term.

If the 5th term and 12th term of an Arithmetic Progression are 30 and 65 respectively, find the sum of its 26 terms.

Find the 10th and nth term of the Geometric sequence   7/2, 7/4, 7/8, 7/16, ...

Find the 15th term of a G.P Whose 8th term is 192 and the common ratio is   2

Find   a   so that a, a+2, a+6 are consecutive terms of a geometric progression.

 If (a-b), (b-c), (c-a) are the consecutive terms of G.P then find (a +b + c)2

 The 7th term of a G.P is eight times of fourth term. What will be the first term when its 5th is 48?

If an amount ₹ 1000 deposited in the bank with annual interest rate 10% interest compounded annually, then find total amount at the end of first, second, third, forth and first years.

Salary of Robin, When his salary is ₹ 5,00,000 per annum for the first year and expected to receive yearly increment of 10%. Now find the Robin salary at staring of 5th year.

The third term of a G.P is 12 and the first term is 48. Find the sum of the first 11 terms

If the second and fourth term of G.P are 8 and 32 respectively. Find the sum of the first nine terms.

if 7 and 189 are the first and fourth term of a G.P respectively, find the sum of the first three terms of the Progression.

Geometric Progression has the second term as 9 and the fourth term as 81. Find the sum of the first four terms.

Find the sum of the geometric series: 4 - 12 + 36 - 108 + ………….. to 10 terms

WEEK 8

LOGARITHM OF NUMBERS

A logarithm of a number is the power to which a given base must be raised to obtain that number. The power is sometimes called the exponent. In other words, if by = x then y is the logarithm of x to base b. For example, if 24 = 16, then 4 is the logarithm of 16 with the base as 2. We can write it as 4 = log 2 = 16.

In mathematics, the logarithm is the inverse function to exponentiation. That means the logarithm of a given number x is the exponent to which another fixed number, the base b, must be raised, to produce that number x. In the simplest case, the logarithm counts the number of occurrences of the same factor in repeated multiplication; e.g. since 1000 = 10 × 10 × 10 = 103, the "logarithm base 10" of 1000 is 3, or log10 (1000) = 3. The logarithm of x to base b is denoted as logb (x), or without parentheses, logb x, or even without the explicit base, log x, when no confusion is possible, or when the base does not matter such as in big O notation.

The logarithm base 10 (that is b = 10) is called the decimal or common logarithm and is commonly used in science and engineering. The natural logarithm has the number e (that is b ≈ 2.718) as its base; its use is widespread in mathematics and physics, because of its simpler integral and derivative. The binary logarithm uses base 2 (that is b = 2) and is frequently used in computer science.

Logarithms were introduced by John Napier in 1614 as a means of simplifying calculations.[1] They were rapidly adopted by navigators, scientists, engineers, surveyors and others to perform high-accuracy computations more easily. Using logarithm tables, tedious multi-digit multiplication steps can be replaced by table look-ups and simpler addition. This is possible because of the fact important in its own right that the logarithm of a product is the sum of the logarithms of the factors:

{\displaystyle \log _{b}(xy)=\log _{b}x+\log _{b}y,}{\displaystyle \log _{b}(xy)=\log _{b}x+\log _{b}y,}

provided that b, x and y are all positive and b ≠ 1. The slide rule, also based on logarithms, allows quick calculations without tables, but at lower precision. The present-day notion of logarithms comes from Leonhard Euler, who connected them to the exponential function in the 18th century, and who also introduced the letter e as the base of natural logarithms.[2]

Logarithmic scales reduce wide-ranging quantities to smaller scopes. For example, the decibel (dB) is a unit used to express ratio as logarithms, mostly for signal power and amplitude (of which sound pressure is a common example). In chemistry, pH is a logarithmic measure for the acidity of an aqueous solution. Logarithms are commonplace in scientific formulae, and in measurements of the complexity of algorithms and of geometric objects called fractals. They help to describe frequency ratios of musical intervals, appear in formulas counting prime numbers or approximating factorials, inform some models in psychophysics, and can aid in forensic accounting.

The concept of logarithm as the inverse of exponentiation extends to other mathematical structures as well. However, in general settings, the logarithm tends to be a multi-valued function. For example, the complex logarithm is the multi-valued inverse of the complex exponential function. Similarly, the discrete logarithm is the multi-valued inverse of the exponential function in finite groups; it has uses in public-key cryptography

Logarithms and Anti-Logarithms

It is not always possible to handle the numbers which are either too large or too small. To make long, tedious and confusing calculations simple, we change the form of the number using logarithms. The changed number can be put into original form by using antilog. Logarithms and Anit-Logarithms are the inverses of each other. Let us study logs and antilog in detail.

Logarithmic Laws and Properties

Theorem 1

The logarithm of the product of two numbers say x, and y is equal to the sum of the logarithm of the two numbers. The base should be the same for both the numbers.

$\log_b (x y) = \log_b x + \log_b y$

Proof: Let $\log_b x = p$ such that $b^p = x$ (i), and

$\log_b y = q$ such that $b^q = y$ (ii)

Multiplying (i), and (ii), we have

$b^p \times b^q = x \times y = b^{(p + q)}$ [from the law of indices]

Taking log on both sides, we have,

$\log_b x y = p + q = \log_b x + \log_b y.$

Theorem 2

The division of the two numbers is the antilog of the difference of logarithm of the two numbers. The base should be the same for both the numbers.

$\log x / y = \log x - \log y$

Proof: Let, $\log_b x = p$ such that $b^p = x$ … (i), and

$\log_b y = q$ such that $b^q = y$ … (ii)

Dividing (i) by (ii), we have

$x/y = b^p/b^q = b^{(p-q)}$ [from the law of indices]

Taking log on both sides, we have,

$\log x/y = p - q = \log x - \log y$

Theorem 3

The logarithm of a number to any other base can be determined by the logarithm of the same number to any given base. Mathematically, the relation is

$\log_a x = \log_b x \times \log_a b$

$\Rightarrow \log_b x = \log_a x / \log_a b$

Proof: Let, $\log_a x = p$, $\log_b x = q$, and $\log_a b = r$. From the definition of logarithms, we have

$a^p = x = b^q$, and $a^r = b$.

$b^q = x$ can be written as $(a^r)^q = a^{rq} = x$.

Since, $a^p = b^q = a^{rq} = x$. Comparing the powers, we have

$p = rq$

or, $\log_a x = \log_a b \times \log_b x$

or, $\log_b x = \log_a x / \log_a b$.

Theorem 4

The logarithm of a number raised to a power is equal to the index of the power multiplied by the logarithm of the number. The base is the same in both the conditions.

$\log_b x^n = n \log_b x$.

Proof: Let $\log_b x = p$ so that $b^p = x$. Raising both sides to power n, we have

$(b^p)^n = x^n \Rightarrow b^{pn} = x^n$

Taking log on both the sides, we have $\log_b x^n = pn$

or, $\log_b x^n = n \log_b x$.

$\log_b (x + y) = \log_b x + \log_b (1 + y/x)$

$\log_b (x - y) = \log_b x + \log_b (1 - y/x)$

Logarithmic Table

It is not always necessary to find the logarithm of a number by mere calculation. We can also use logarithm table to find the logarithm of a number. The logarithm of a number comprises of two parts. The whole part is the characteristics and the decimal part is the mantissa.

How to Use Log Table?: Step-By-Step Process with Example

To find the log value of a number using the log table, you must understand the process of reading the log table. We have provided a step-by-step process to find the values using an example:

Step 1: Identify the table. For different bases, a different log table is used. The table provided above is for base 10. So, you can find the log value of a number to the base 10 only. To find natural logarithms or binary logarithms, you will have to use a different table.

Step 2: Find out the integer and decimal parts of the given number. Suppose we want to find the log value of n = 18.25. So, first of all, we separate the integer and decimal.

Integer Part: 18
Decimal Part: 25

Step 3: Come to the common log table and look for the cell value at the following intersections:

The row labelled with first two digits of n
Column header with the third digit of n

⇒ In this example, log10(18.25) → row 18, column 2 → cell value 2601. So, the value obtained is 2601.

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20

Step 4: Always use the common logarithm table with a mean difference. Now again go to row number 18 and column number 5 (fourth digit of n) in the mean difference table.

⇒ In this example, log₁₀(18.25) → row 18, mean difference column 5 → cell value 12. Write down the corresponding value which is 12.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	4	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19

Mean Difference

Step 5: Add both the values obtained in step 3 and step 4.
That is 2601 + 12 = 2613.
Step 6: Find the characteristic part. By trial and error, find the integer value of *p* such that a^p < n and a^{p+1} > n. Here a is the base and p is the characteristic part. For common (base 10) logs, just count the number of digits left of the decimal and subtract one.
So, Characteristic part = (number of digits to the left of the decimal   1).
In this example, characteristic = 2   1
= 1
Step 7: Combine both the Characteristic and the Mantissa part and you will get the final value which is 1.2613.
So, log$_{10}$ (18.25) = 1.2613

Example   1:   Use   logs   to evaluateN=647·32×0.000001478.473×64N=647·32×0.000001478.473×64

Solution: Our approach consists of four steps:

convert the expression for N into logs

evaluate those logs using a log table

thus determine log N

calculate antilog of log N

Thus,

logN=(647·32×0.000001478.473×64)=log(647·32)+log(0.00000147)−log(8.473)−log(64)log N=(647·32×0.000001478.473×64)=log (647·32)+log (0.00000147)−log (8.473)−log (64)

Now, note that (use log tables):

$$647.32 = 6.4732 \times 10^2 \Rightarrow \log(647.32) = 2 + \log(6.4732) = 2 + 0.8111 = 2.8111$$
$$0.00000147 = 1.47 \times 10^{-6} \Rightarrow \log(0.00000147) = -6 + \log(1.47) = \overline{6}.1673$$
$$8.473 = 8.473 \times 10^0 \Rightarrow \log(8.473) = 0 + 0.9280 = 0.9280$$
$$64 = 6.4 \times 10^1 \Rightarrow \log(64) = 1 + \log(64) = 1.8062$$

Thus,

$$\log N = 2.8111 + \overline{6}.1673 - 0.9280 - 1.8062 = 2.8111 + (-6 + 0.1673) - 0.9280 - 1.8062 = -5.7558 = -5 - 0.7558 = (-5 - 1) + 1 - 0.7558 = -6 + 0.2442 = \overline{6} \cdot 2442$$

Note the last couple of steps carefully. Now, we have the characteristic and mantissa of log N.

We thus find antilog (mantissa) = antilog (0.2442) = 1.7547 {from the antilog tables}:

$$\Rightarrow N = 1.7547 \times 10^{-6} = 0.0000017547$$

Verify this using a calculator. Note that N was calculated without any actual multiplication and division – operations much more cumbersome than addition or subtraction.

Example 2: Find the value of $\sqrt[5]{0.00000165}$

Solution: Let $N = \sqrt[5]{0.00000165}$

$$\Rightarrow \log N = \tfrac{1}{5}\log(0.00000165) = \tfrac{1}{5}(-6 + \log 1.65) \quad (\text{why?})$$

Using the log tables, we have:

$$\log N = \tfrac{1}{5}(-6 + .2175) = -1.2 + 0.0435 = -2 + 0.8435 = \overline{2} \cdot 8435$$

Thus,

$N = \text{antilog}(0.8435)10^{-2} = 6.974 \times 10^{-2} = 0.06974$

Example 3:

How many digits will there be in 875^{16}?

Solution: Let $N = 875^{16}$. We have:

$$\Rightarrow \log N = 16\log(875) = 16\log\{\log(8.75 \times 10^2)\} = 16(2 + \log 8.75) = 16 \times 2.9420 = 47.0721$$

$$\Rightarrow N = \underbrace{\text{antilog}(0.0721)}_{\text{Contains 1 digit before the decimal}} \times \underbrace{10^{47}}_{\text{has 47 zeroes}}$$

Thus, N will contain 48 digits. Note that we do not need to calculate the antilog of 0.0721 to arrive at this final answer.

Example 4: Evaluate $\dfrac{(33049)^4}{\sqrt[3]{22 \times 70}}$

Solution: If we let N denote this expression, we have

$$\log N = 4\log(33049) - \tfrac{1}{3}\log(22 \times 70) = 4\log 330 - 4\log 49 - \tfrac{1}{3}\log 22 - \tfrac{1}{3}\log 70$$

Noticing that $330 = 11 \times 3 \times 10, 49 = 7^2, 22 = 2 \times 11$ and $70 = 7 \times 10$, we have

$$\log N = 4(1 + \log 3 + \log 11) - 8\log 7 - \tfrac{1}{3}(\log 2 + \log 11) - \tfrac{1}{3}(1 + \log 7) = \tfrac{11}{3} - \tfrac{1}{3}\log 2 + 4\log 3 - \tfrac{25}{3}\log 7 + \tfrac{11}{3}\log 11 = \tfrac{11}{3} - \tfrac{1}{3}(0.3010) + 4(0.4771) - \tfrac{25}{3}(0.8451) + \tfrac{11}{3}(1.0414)$$

$$= 3.6666 - 0.1003 + 1.9084 - 7.0425 + 3.8185 = 2.2507$$

Thus,

$$N = \text{antilog}(0.2507) \times 10^2 = 178.1148$$

The final result obtained this way may differ somewhat from the actual result due to rounding-off errors along the way.

Example 5:

Solve the following system of equations for x and y :

$2^{x+y}=6^y$

$3^x=3\cdot2^{y+1}$

Solution: We have

$(x+y)\log 2=y\log 6=y(\log 2+\log 3)$

$\Rightarrow x\log 2=y\log 3\cdots(1)$
and, $x\log 3=\log 3+(y+1)\log 2$

$\Rightarrow (x-1)\log 3=(y+1)\log 2\ldots(2)$

Using (1) and 2, we have

$x\log 2=((x-1)\log 3\log 2-1)\log 3$

Using log 2 = a, log 3 = b for convenience, we have

$ax=(b(x-1)a-1)b$

$\Rightarrow abx=bax-ba-1$

$\Rightarrow (ba-ab)x=1+ba=a+ba$

$\Rightarrow (b^2-a^2ab)x=a+ba$

$\Rightarrow x=\dfrac{b}{b-a}=\dfrac{\log 3}{\log 3-\log 2}\approx\dfrac{\cdot4771}{\cdot1761}\approx2.71$

Similarly, $y\approx1.71$.

Example 6: Prove that the following relation is correct for three numbers a, b and c (assume that all the terms are well-defined):

$\log_a b=\log_c b\log_c a$

Solution: We suppose the following:

$\log_c b = x, \log_c a = y, \log_a b = z$ logcb=x,logca=y,logab=z

Thus,

$$\begin{cases} c^x = b, \\ c^y = a \end{cases} a^z = b \Rightarrow (c^y)^z = b = c^x \Rightarrow c^{yz} = c^x \Rightarrow yz = x \Rightarrow z = \frac{x}{y} \Rightarrow \log_a b = \frac{\log_c b}{\log_c a}$$ {cx=b,cy=aaz=b⇒(cy)z=b=cx⇒cyz=cx⇒yz=x⇒z=yx⇒logab=logcblogca

This relation enables us to change bases. For example, suppose that we have to calculate $\log_{32} 2048$ log322048. We can change the base to 2, as follows: $\log_{32} 2048 = \frac{\log_2 2048}{\log_2 32} = \frac{11}{5} = 2.2$ log322048=log22048log232=115=2.2

An approximation is **anything that is similar, but not exactly equal, to something else**. A number can be approximated by rounding. A calculation can be approximated by rounding the values within it before performing the operations .

Approximate Numbers and Significant Figures
An exact number is one that has no uncertainty. An example is the number of tires on a car (exactly 4) or the number of days in a week (exactly 7). An approximate number is one that does have uncertainty. A number can be approximate for one of two reasons:
The number can be the result of a measurement. For example a certain instrument capable of measuring to the nearest 0.1 cm may measure the length of a certain bolt to be 8.6 cm. A better quality instrument capable of measuring to the nearest 0.001 cm may give the length of the same bolt to be 8.617 cm. This new number is less approximate but is still not exact.

Certain numbers simply cannot be written exactly in decimal form. Many fractions and all irrational numbers fall into this category. For example the fraction 1/3 is approximately but not exactly equal to 0.333 and the irrational number is approximately but not exactly equal to 1.73.

When we state that the measured length of the bolt is 8.6 cm then we actually mean that the value is closer to 8.6 cm than it is to 8.5 cm or 8.7 cm. The true length could be anywhere in the gray area shown here:

And when we state that the more accurate instrument gave the length of the bolt to be approximately 8.617 cm then we mean that the value is closer to 8.617 cm than it is to 8.616 cm or 8.618 cm. The true length could still be anywhere in the gray area shown here:

If someone told us that they used this same instrument and got a reading of 8.61712345 would we believe it? No way! Adding just one atom to the end of the bolt would cause the last digit to change! The extra digits are meaningless and are said to be insignificant. To claim that they are correct is nonsense.

Significant Digits or Figures

Definitions:

In an approximate number the leftmost digit is said to be the most significant digit and the rightmost digit is the least significant digit. All the digits in the number are significant digits (also known as significant figures or sig. figs.) with one exception: if the digit is a zero that is used just to locate the decimal point then it is not significant.

The accuracy of an approximate number is given by the number of significant digits in it.

The precision of an approximate number is given by the position of the rightmost significant digit.

Examples:
The approximate number 8.617 has 4 significant digits. The digit 8 is the most significant digit and the digit 7 is the least significant digit.

The number 1.23, the number 0.000123 and the number 123000000 all have an accuracy of 3 sig. figs. All the zeros are used simply to locate the decimal point.

The number 1.23 has a precision of 0.01, the number 0.000123 has a precision of 0.000001 and the number 123000000 has a precision of 1000000.

The number 1.023, the number 0.01023 and the number 1002000 all have 4 sig. figs. (The zeros shown in red are used simply to locate the decimal point and don't count as sig. figs.)

Notes: The precision of a measuring instrument is the difference between the two closest readings that the instrument can differentiate. For example the above instruments had a precision of 0.1 cm and 0.001 cm.

Precision and accuracy are not the same thing. Accuracy has to do with the quality (and cost!) of the measurement. For example if your instrument has a precision of 1 centimeter then that may not be very accurate if that instrument is designed to measure distances between objects on your desk but it would be very accurate if it was designed to measure the distances between the planets.

Rounding
We saw above that if an instrument capable of measuring to the nearest 0.1 gave a measured value of 8.6 then the true value could be anywhere in the gray area:

Rounding is exactly the same idea but reversed. Rounding to the nearest 0.1 means to replace any number in the gray area (from 8.55 to 8.65) by 8.6. In general, rounding is done like this:

Rounding: When rounding to a certain place value then all digits to the right of that place are dropped. If the first dropped digit is 0, 1, 2, 3, or 4 then the least significant digit kept is not changed. (This is called rounding down.) If the first dropped digit is 5, 6, 7, 8 or 9 then the least significant digit kept is increased by 1. (This is called rounding up.)

You can round to either a given decimal place or to a given number of sig. figs. Here are some examples of rounding to 2 decimal places (the dropped digits are shown in red):
the rounding   the rule used
4.384 → 4.38   first dropped digit is a 4 so round down
4.3851 → 4.39   first dropped digit is a 5 so round up
0.00043851 → 0.00   first dropped digit is a 0 so round down

Here are some examples of rounding to 3 sig. figs (the dropped digits are shown in red):
the rounding   the rule used
4.384 → 4.38   first dropped digit is a 4 so round down
43851 → 43900   first dropped digit is a 5 so round up
0.00043851 → 0.000439   first dropped digit is a 5 so round up